16.90 Spring 2012
Midterm Exam Solution

Question 1

In this problem, we consider two ODE initial value problem

$$\frac{du}{dt} = f(u), \qquad u(t=0) = u_0. \tag{1}$$

and

$$\frac{dv}{dt} = u(t) - v(t), \qquad v(t=0) = v_0. \tag{2}$$

Note that the second equation has a term that depends on the solution $u(t)$ of the first equation.

(a) Consider that Equation (1) is solved using a stable scheme with fifth order local accuracy, with time step size Δt. The numerical solution at time $n\,\Delta t$ is denoted as $\hat{u}_n$. At what rate does the error in $\hat{u}_n$ (compared to the exact solution $u(n\,\Delta t)$) decrease as Δt decreases?

(b) Given the numerical solution $\hat{u}_n$ in Part (a), consider two numerical methods for solving Equation (2):

(i)

$$\frac{\hat{v}^{n+1} - \hat{v}^n}{\Delta t} = \hat{u}^n - \hat{v}^{n+1}$$

(ii)

$$\frac{\hat{v}^{n+1} - \hat{v}^{n-1}}{2\Delta t} = \hat{u}^n - \hat{v}^n$$

What is the local order of accuracy of each scheme? What is the global order of accuracy of each scheme?

(c) Are these schemes explicit or implicit? How do you determine their amplification factors? When are they stable / unstable?