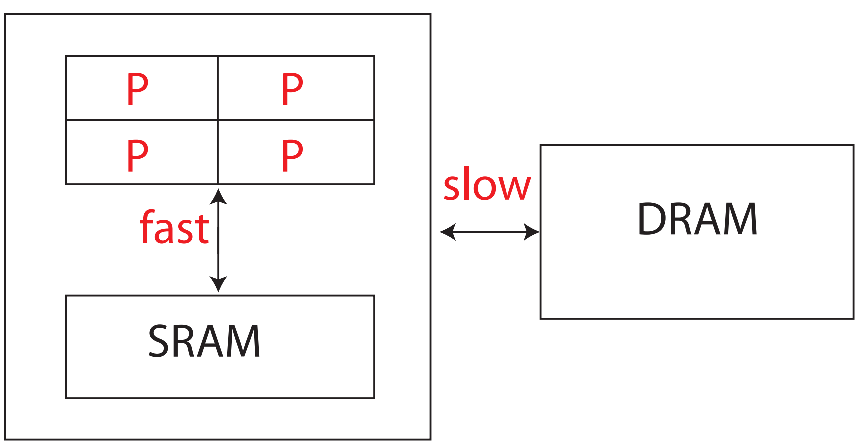
# Lecture 24: Parallel Processor Architecture & Algorithms

## Processor Architecture

Computer architecture has evolved:

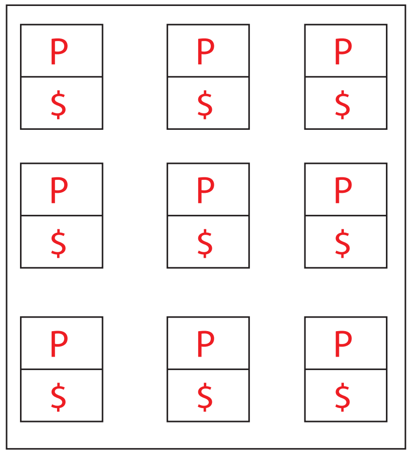
- Intel 8086 (1981): 5 MHz (used in first IBM PC)
- Intel 80486 (1989): 25 MHz (became i486 because of a court ruling that prohibits the trademarking of numbers)
- Pentium (1993): 66 MHz
- Pentium 4 (2000): 1.5 GHz (deep $\approx$ 30-stage pipeline)
- Pentium D (2005): 3.2 GHz (and then the clock speed stopped increasing)
- Quadcore Xeon (2008): 3 GHz (increasing number of cores on chip is key to performance scaling)

Processors need data to compute on:



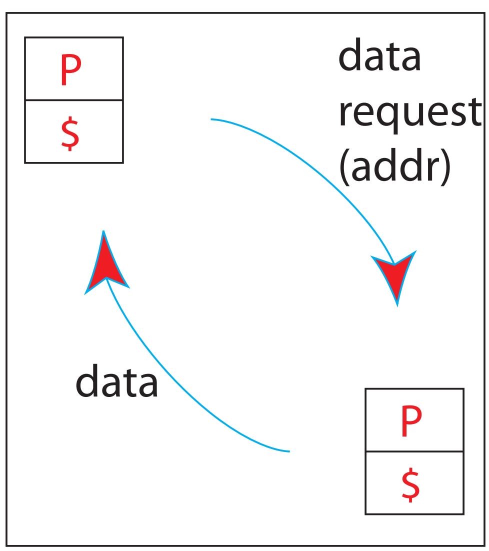

Problem: SRAM cannot support more than $\approx$ 4 memory requests in parallel.

$: cache P: processor




Most of the time program running on the processor accesses local or "cache" memory

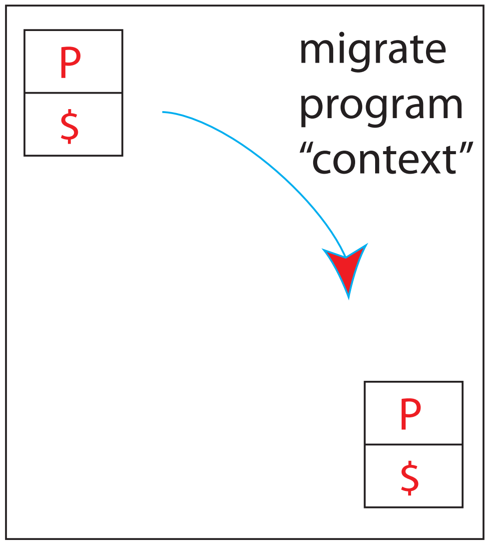
Every once in a while, it accesses remote memory:




Round-trip required to obtain data

## Research Idea: Execution Migration

When program running on a processor needs to access cache memory of another processor, it migrates its "context" to the remote processor and executes there:




One-way trip for data access

Context $= \underbrace{\text{ProgramCounter} + \text{RegisterFile} + \dots}_{\text{fewKbits}}$ (can be larger than data to be accessed)

Assume we know or can predict the access pattern of a program

$m_1, m_2, \dots, m_N$ (memory addresses)

$p(m_1), p(m_2), \dots p(m_N)$ (processor caches for each $m_i$)

### Example

$p_1\ p_2\ p_2\ p_1\ p_1\ p_3\ p_2$

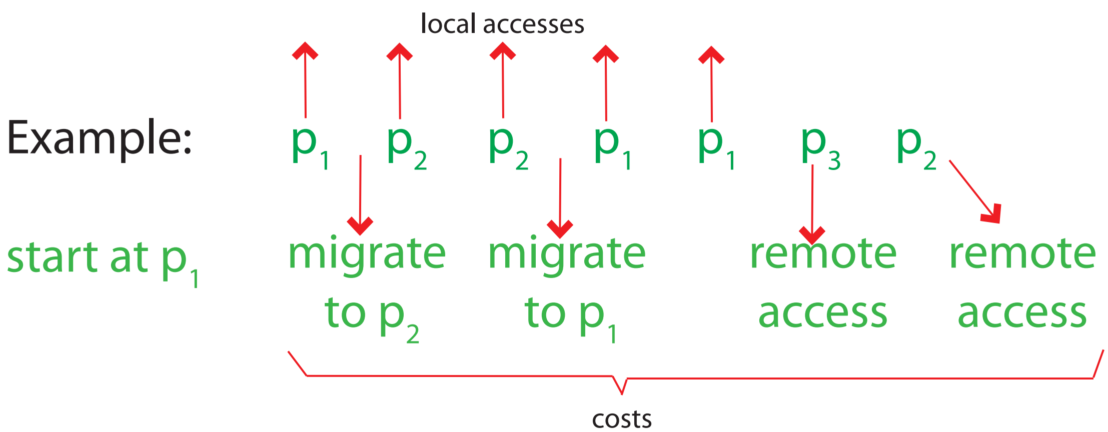
$\text{cost}_{\text{mig}}(s, d) = \text{distance(s, d)} + \text{L}$ $\leftarrow$ load latency $L$ is a function of context size

$\text{cost}_{\text{access}}(s, d) = 2 * \text{distance(s, d)}$

if $s == d$, costs are defined to be 0

### Problem

Decide when to migrate to minimize total memory cost of trace For example:




What can we use to solve this problem?
Dynamic Programming!

### Dynamic Programming Solution

Program at $p$, initially, number of processors $= Q$

### Subproblems?

Define $DP(k, p_i)$ as cost of optimal solution for the prefix $m_1, \ldots, m_k$ of memory accesses when program starts at $p_1$ and ends up at $p_i$.

$$DP(k+1, p_j) = \begin{cases} DP(k, p_j) + \mathrm{cost_{access}}(\mathrm{p_j}, \mathrm{p}(\mathrm{m_{k+1}})) & \text{if } p_j \neq p(m_{k+1}) \\ MIN_{i=1}^{Q}(DP(k, p_i) + \mathrm{cost_{mig}}(\mathrm{p_i}, \mathrm{p_j})) & \text{if } p_j = p(m_{k+1}) \end{cases}$$

### Complexity?

$$O(\underbrace{N \cdot Q}_{\text{no.of subproblems}} \cdot \underbrace{Q}_{\text{cost per subproblem}}) = O(NQ^2)$$

My research group is building a 128-processor Execution Migration Machine that uses a migration predictor based on this analysis.

# Lecture 24: Research Areas and Beyond 6.006

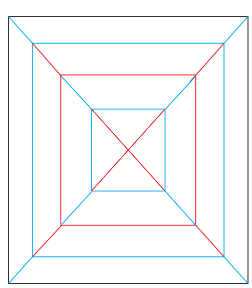
## Erik's Main Research Areas

- computational geometry [6.850]
  - geometric folding algorithms [6.849]
  - self-assembly
- data structures [6.851]
- graph algorithms [6.899]
- recreational algorithms [SP.268]
- algorithmic sculpture

## Geometric Folding Algorithms: [6.849], Videos Online

Two aspects: design and foldability

- design: algorithms to fold any polyhedral surface from a square of paper [Demaine, Demaine, Mitchell (2000); Demaine & Tachi (2011)]
  - bicolor paper $\implies$ can 2-color faces
  - OPEN: how to best optimize "scale-factor"
  - e.g. best $n \times n$ checkerboard folding — recently improved from $\approx n/2 \rightarrow \approx n/4$
- foldability: given a crease pattern, can you fold it flat
  - NP-complete in general Bern & Hayes (1996)
  - OPEN: $m \times n$ map with creases specified as mountain/valley [Edmonds (1997)]
  - just solved: $2 \times n$ Demaine, Liu, Morgan (2011)
  - hyperbolic paraboloid [Bauhaus (1929)] doesn't exist [Demaine, Demaine, Hart, Price, Tachi (2009)]



- understanding circular creases
- any straight-line graph can be made by folding flat & one straight cut [Demaine, Demaine, Lubiw (1998); Bern, Demaine, Eppstein, Hayes (1999)]

## Self-Assembly

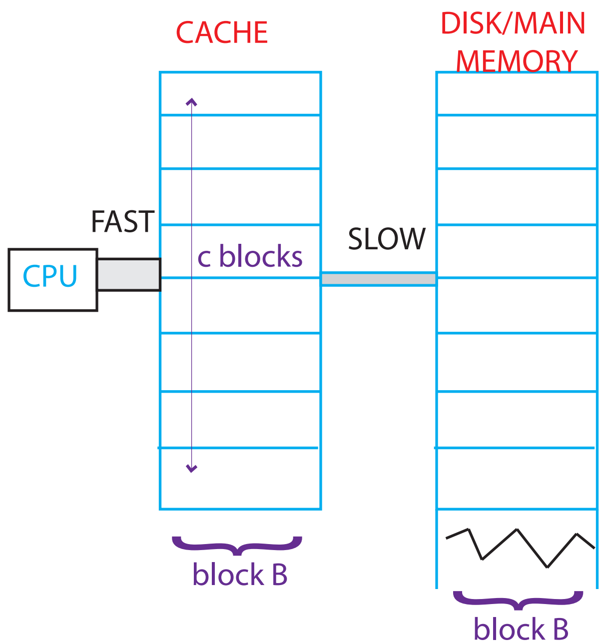
Geometric model of computation

- glue e.g. DNA strands, each pair has strength
- square tiles with glue on each side
- Brownian motion: tiles/constructions — stick together if $\sum$glue strengths $\geq$ temperature
- can build $n \times n$ square using $O\left(\frac{\lg n}{\lg \lg n}\right)$ tiles [Rothemund & Winfree 2000] or using $O(1)$ tiles & $O(\lg n)$ "stages" algorithmic steps by the bioengineer [Demaine, Demaine, Fekete, Ishaque, Rafalin, Schweller, Souvaine (2007)]
- can replicate $\infty$ copies of given unknown shape using $O(1)$ tiles and $O(1)$ stages [Abel, Benbernou, Damian, Demaine, Flatland, Kominers, Schweller (2010)]

## Data Structures: [6.851], Videos Next Semester

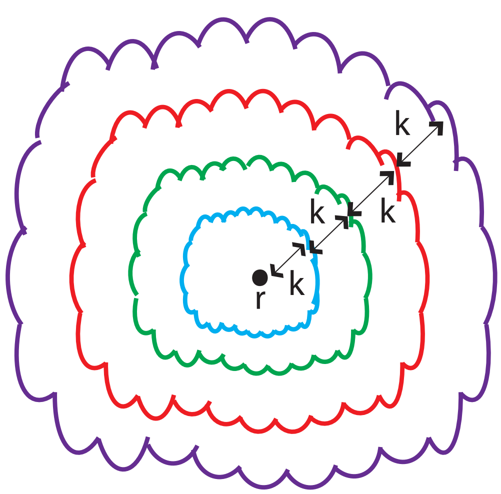
There are 2 main categories of data structures

- Integer data structures: store $n$ integers in $\{0, 1, \cdots u-1\}$ subject to insert, delete, predecessor, successor (on word RAM)
  - hashing does exact search in $O(1)$
  - AVL trees do all in $O(\lg n)$
  - $O(\lg \lg u)$/op van Emde Boas
  - $O\left(\frac{\lg n}{\lg \lg u}\right)$/op fusion trees: Fredman & Willard
  - $O\left(\sqrt{\frac{\lg n}{\lg \lg n}}\right)$/op min of above
- Cache-efficient data structures
  - memory transfers happen in blocks (from cache to disk/main memory)
  - searching takes $\Theta(\log_B N)$ transfers (vs. $\lg n$ )
  - sorting takes $\Theta\left(\frac{N}{B} \log_C \frac{N}{B}\right)$ transfers
  - possible even if you don't know B & C !




(Almost) Planar Graphs: [6.889], Videos Online

- Dijkstra in $O(n)$ time [Henzinger, Klein, Rao, Subramanian (1997)]
- Bellman-Ford in $O\left(\frac{n \lg^2 n}{\lg \lg n}\right)$ time [Mozes & Wolff-Nilson (2010)]
- Many problems NP-hard, even on planar graphs. But can find a solution within $1 + \varepsilon$




factor of optimal, for any $\epsilon$ [Baker 1994 & Others]:

- run BFS from any root vertex $r$
- delete every $k$ layers
- for many problems, solution messed up by only $1 + \frac{1}{k}$ factor ($\implies k = \frac{1}{\varepsilon}$)
- connected components of remaining graph have $< k$ layers. Can solve via DP typically in $\approx 2^k \cdot n$ time

## Recreational Algorithms

- many algorithms and complexities of games [some in SP.268 and our book *Games, Puzzles & Computation* (2009)]
- $n \times n \times n$ Rubik's Cube diameter is $\Theta\left(\frac{n^2}{\lg n}\right)$ [Demaine, Demaine, Eisenstat, Lubiw, Winslow (2011)]
- Tetris is NP-complete [Breukelaar, Demaine, Hohenberger, Hoogeboom, Kosters, Liben-Nowell (2004)]
- balloon twisting any polyhedron [Demaine, Demaine, Hart (2008)]
- algorithmic magic tricks

**Algorithms Classes at MIT: (post 6.006)**

- 6.046: Intermediate Algorithms (more advanced algorithms & analysis, less coding)
- 6.047: Computational Biology (genomes, phylogeny, etc.)
- 6.854: Advanced Algorithms (intense survey of whole field)
- 6.850: Geometric Computing (working with points, lines, polygons, meshes, ...)
- 6.849: Geometric Folding Algorithms origami, robot arms, protein folding, ...
- 6.851: Advanced Data Structures (sublogarithmic performance)
- 6.852: Distributed Algorithms (reaching consensus in a network with faults)
- 6.853: Algorithmic Game Theory (Nash equilibria, auction mechanism design, ...)
- 6.855: Network Optimization (optimization in graph: beyond shortest paths )
- 6.856: Randomized Algorithms (how randomness makes algorithms simpler & faster)
- 6.857: Network and Computer Security (cryptography)

**Other Theory Classes:**

- 6.045: Automata, Computability, & Complexity
- 6.840: Theory of Computing
- 6.841: Advanced Complexity Theory
- 6.842: Randomness & Computation
- 6.845: Quantum Complexity Theory
- 6.440: Essential Coding Theory
- 6.441: Information Theory

# Top 10 Uses of 6.006 Cushions

10. Sit on it: guaranteed inspiration in constant time
    (bring it to the final exam)
9. Frisbee (after cutting it into a circle)*
8. Sell as a limited-edition collectible on eBay
   (they'll probably never be made again—at least $5)
7. Put two back-to-back to remove branding*
   (so no one will ever know you took this class)
6. Holiday conversation starter… and stopper
   (we don't recommend re-gifting)
5. Asymptotically optimal acoustic paneling
   (for practicing piano & guitar fingering DP)
4. Target practice for your next LARP*
   (Live Action Role Playing)
3. Ten years from now, it might be all you'll remember about 6.006
   (maybe also this top ten list)
2. Final exam cheat sheet*
1. *Three words:* OkCupid profile picture