

Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.685 Electric Machines

Problem Set 2

Issued September 9, 2013

Due September 18, 2013

Problem 1: A two-coil rotary system has self- and mutual inductances of this form:

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} L_1 & M \cos \theta \\ M \cos \theta & L_2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

Assume values of the inductances to be $L_1 = L_2 = 100\text{mHy}$ and $M = 80\text{mHy}$.

Note that this system has coupling coefficient less than one ($M^2 < L_1 L_2$). It is rotated at a constant speed of 3,600 RPM (60 Hz). Current in Coil 1 is held constant $i_1 = I_1 = 5\text{A}$. It has been on long enough to be in steady state operation.

Find and plot the answers to the following questions. Your plots should show two complete rotations of the system. Time origin is not important.

- What torque does this system produce?
- What is the voltage (ignore resistance) at the terminals of Coil 1?

under these three cases:

- Current in Coil 2 is held constant ($i_2 = I_2 = 5\text{A}$)
- *Flux* linked by coil 2 is constrained to be zero ($\lambda_2 = 0$). Note: this is what would be approximately true were the terminals of Coil 2 to be directly shorted and the resistance of Coil 2 is small).
- Coil 2 is shorted through a resistor so that the total resistance (including the coil itself) is 20Ω .

Finally, estimate the *average* value of torque for the last case: in which $I_1 = 5\text{A}$ and Coil 2 is shorted through a 20Ω resistor, over a range of rotor speed between 1,500 and 2,500 RPM.

Problem 2: This problem is related to a 'coil gun' problem we will be doing with the freshmen this year. The situation is shown in Figure 1. A permanent magnet can move through a plastic tube. For our purposes we will assume the tube is frictionless. A coil of wire is wrapped around the tube.

In an experiment, the permanent magnet is made to move through the tube with a velocity of 5 m/S. The coil is open circuited and its voltage is measured to be approximately what is shown in Figure 2.

As it turns out, that voltage may be expressed in the form:

$$v(t) = Ate^{-\alpha t}$$

where $A = 1000e \approx 2,718\text{V/s}$ and $\alpha = 500\text{s}^{-1}$ Note that, in the figure and in this expression we set $t = 0$ to the point in time at which the magnet is right in the center of the coil, which is when the voltage goes through zero.

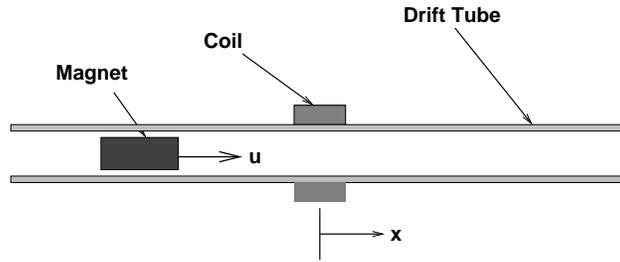


Figure 1: Moving Magnet Problem

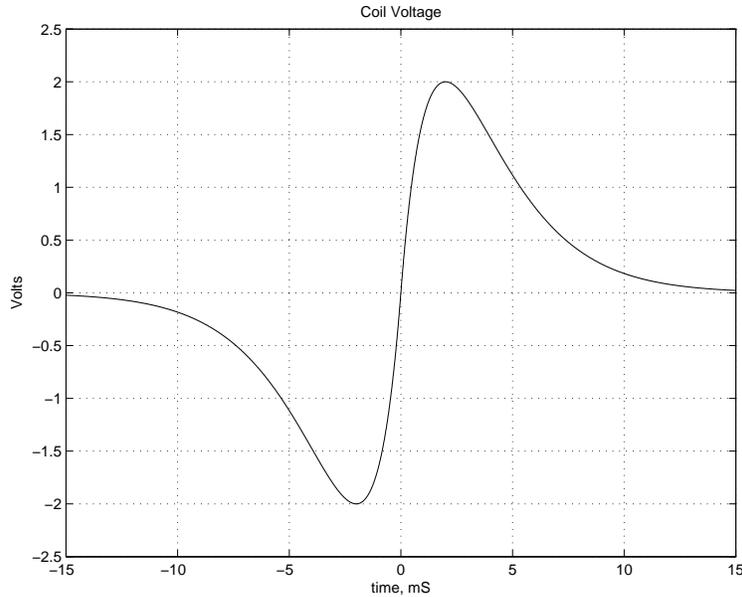


Figure 2: Induced Voltage in Coil

1. If the magnet is held stationary in the tube and coil current is fixed at $i = 1A$, what is force on the magnet as a function of position? Assume $x = 0$ is the middle of the coil.
2. The magnet weighs 50 grams. The magnet starts at $x = 1cm$ when the coil current is stepped from zero to some value I_0 . The objective is to shoot the magnet to a height of 1 meter. What value should I_0 have?

Problem 3: This problem relates to magnetic lifting systems such as are used in so-called “attractive levitation” schemes for high speed ground transport. See Figure 3. An example of a transportation system that uses such a scheme (Not quite this embodiment of it!) is the German built Transrapid system. The stationary element is the fixed ‘rail’. Mounted on and supporting the car is a ferromagnetic yoke and a current carrying coil. When the coil is excited it produces magnetic fields that attract the yoke to the rail. A section of the system is shown in a little more detail in Figure 4.

This would be only one of several magnets supporting a ‘car’. Assume that this magnet is supporting about 5 metric tons, which implies about 50 kN of required lift force. Dimensions and other data are as follows:

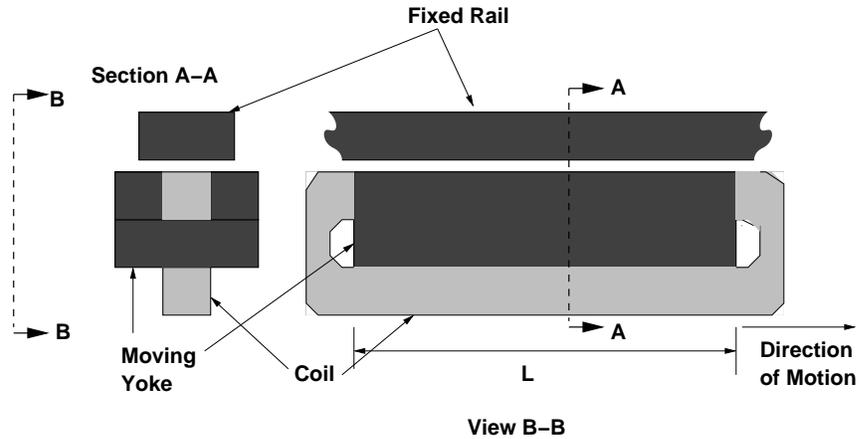


Figure 3: Suspension Magnet

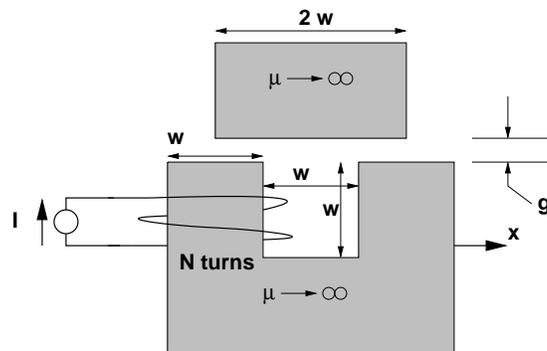


Figure 4: Suspension Magnet Details

Support Force Required	F	50,000 N
Magnet Length	L	1.0 m
Pole width	w	0.05 m
Relative Motion Gap	g	0.015 m

Note that the pole width is w and we assume the coil dimensions are $w \times w$. The rail width is such that, when the yoke is centered, each side has an overlap of $\frac{1}{2}w$.

1. With the yoke centered and providing the nominal support force, what is the required number of ampere-turns in the exciting coil?
2. How does this required current change with yoke lateral position x ? Calculate and plot the required current as a function of position, ranging from $-w/4$ to $+w/4$.
3. If the car moves laterally (direction x), this system provides a restoring force. Find that lateral force as a function of x and exciting current NI .
4. Now combine these two parts and, assuming that current is controlled to maintain a 1 cm relative motion gap, find and sketch (or have MATLAB plot) restoring force as a function of lateral position.

NOTE: You may find a reading of Chapter 2 of the notes, on magnetic circuits, will make some of these calculations easier.

MIT OpenCourseWare
<http://ocw.mit.edu>

6.685 Electric Machines
Fall 2013

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.