

Massachusetts Institute of Technology
 Department of Electrical Engineering and Computer Science
 6.685 Electric Machines

Problem Set 7 Solutions

October 23, 2013

Problem 1: Induction Motor

The story here is told by the script, attached, that does all of the heavy lifting. Look at Figure 1 that shows the equivalent circuit for the machine. Note that the core loss is represented by a resistive element. More on that in a few lines.

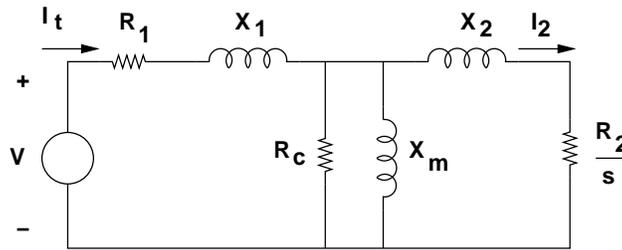


Figure 1: Induction Motor Equivalent Circuit

For each point of operation, the impedance of the core is:

$$Z_c = R_c || jX_m$$

And then the air-gap impedance is:

$$Z_{ag} = Z_c || \left(jX_2 + \frac{R_2}{s} \right)$$

Total machine impedance is:

$$Z_t = R_1 + jX_1 + Z_{ag}$$

Voltage across the core resistance and magnetizing reactance is:

$$V_c = V \frac{Z_{ag}}{Z_t}$$

Terminal current is:

$$I_t = \frac{V}{Z_t}$$

And rotor current is found from a simple current divider:

$$I_2 = I_t \frac{Z_c}{Z_c + jX_2 + \frac{R_2}{s}}$$

Air-gap power is:

$$P_{ag} = 3|I_2|^2 \frac{R_2}{s}$$

Terminal input power is:

$$P_{in} = 3\text{Re}\{VI_t^*\}$$

Now: there are a few ad-hoc and heuristic ways we handle certain loss elements. One is 'stray' loss, which may be estimated by considering space harmonic parasitic circuits (you will see this in Problem Set 8). For now, it is appropriate to simply treat stray loss as drag. Thus mechanical output power is:

$$P_m = P_{ag} (1 - s) (1 - f_s)$$

where f_s is the 'stray loss factor', which here is 2.5%.

Now: to handle core loss, we treat it as a resistance. The issue is that this is not actually a linear element, and variations of frequency and voltage across the core will affect the value of that resistance. For our purposes, we see that core loss can be represented as:

$$P_c = \frac{3|V_c|^2}{R_c} = \frac{3V_0^2}{R_c} \left(\frac{f}{f_0}\right)^2 \left(\frac{B}{B_0}\right)^2 = \frac{3V_0^2}{R_0} \left(\frac{f}{f_0}\right)^{1.8} \left(\frac{B}{B_0}\right)^{2.2}$$

Where the quantities subscripted '0' are base elements. The value to be used for core resistance is then:

$$R_c = R_0 \left(\frac{f}{f_0}\right)^{-0.2} \left(\frac{B}{B_0}\right)^{0.2}$$

Then, noting that

$$\frac{V}{V_0} = \frac{f}{f_0} \frac{B}{B_0}$$

We have a value to use for core resistance:

$$R_c = R_0 \left(\frac{V}{V_0}\right)^{0.2} \left(\frac{f}{f_0}\right)^{-0.4}$$

Since the voltage across the core element is a function of voltage, we can use an iterative approach: a simple gaussian iteration to find the actual core voltage. Some value for core resistance is used to find voltage, that voltage is used to find a new value for core resistance, and the process is repeated until two successive values of resistance are within some small tolerance. For details on this, see the script that is appended. Torque-speed and power-speed curves for Problem 1 are shown in Figures 2 and 3.

To find data on operation over a range of mechanical load, we first find the values of slip for the lower and upper values of that load and cross plot, as is shown in Figure 4.

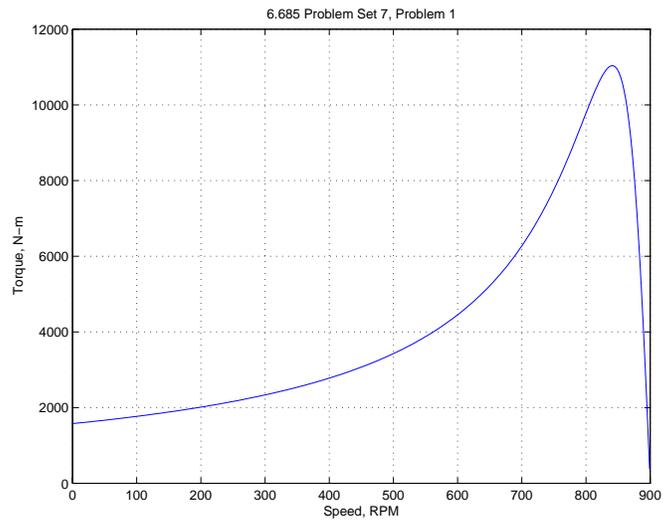


Figure 2: Induction Motor Torque vs. Speed, Base Frequency

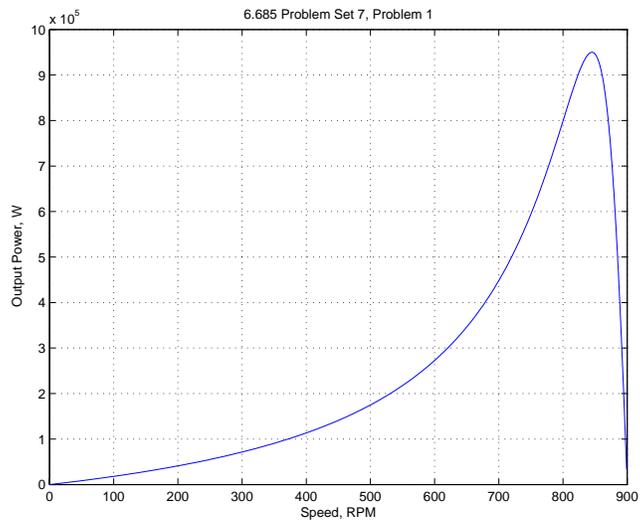


Figure 3: Induction Motor Power vs. Speed, Base Frequency

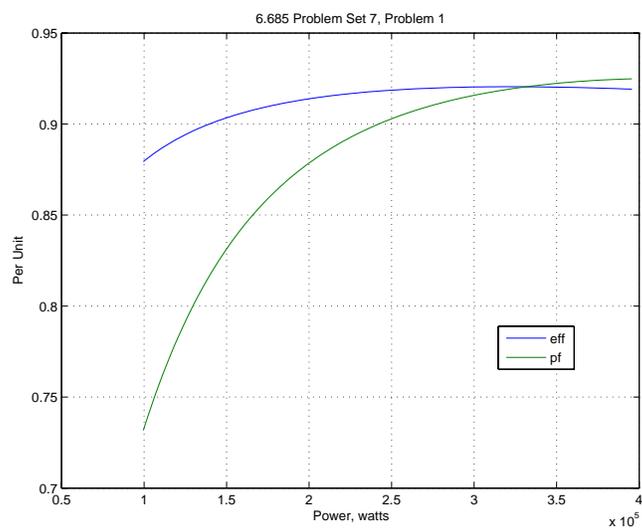


Figure 4: Efficiency and Power Factor, Base Frequency, Varying Load

Problem 2 This problem asks for operation over a range of frequencies. The first part of this is similar to the first part of the first problem, but requires that the reactances be corrected for actual frequency:

$$\begin{aligned}
 X_1 &= X_{1b} \frac{f}{f_0} \\
 X_2 &= X_{2b} \frac{f}{f_0} \\
 X_m &= X_{mb} \frac{f}{f_0}
 \end{aligned}$$

Core resistance is handled as in the first problem. Torque-speed curves for the several frequencies cited are shown in Figure 5

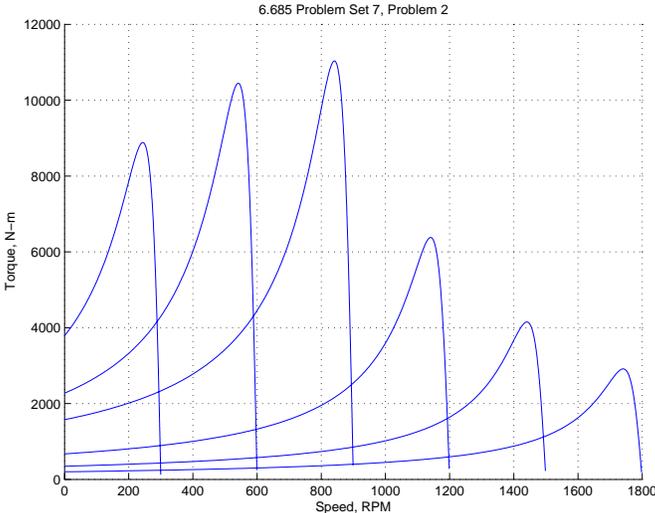


Figure 5: Torque-Speed Curves at Several Frequencies

Finally, all of the techniques described are used to estimate efficiency and machine power factor for a volts/Hz drive application. Note the odd looking kink at the point where the system goes from volts/Hz to constant voltage.

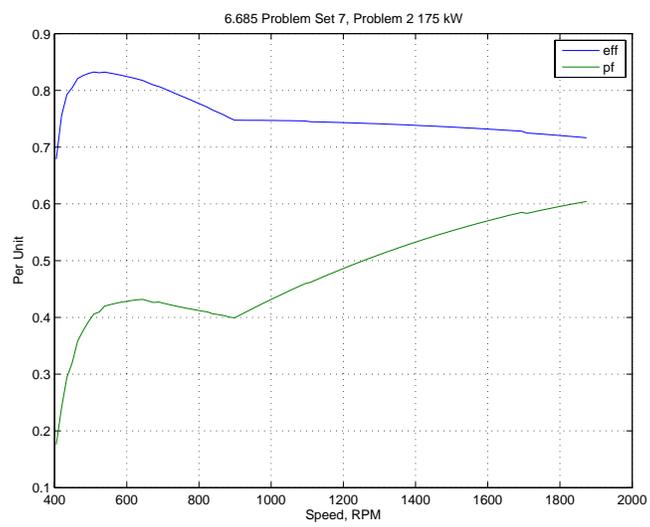


Figure 6: Adjustable Speed Drive Performance

Problem 3 This is just an exercise in using MATLAB to do a complex calculation and plotting the result. For a bar with width w and height h , the impedance is:

$$Z_b = \frac{1}{w} \frac{1+j}{\sigma \delta} \coth(1+j) \frac{h}{\delta}$$

where the skin depth is:

$$\delta = \sqrt{2\omega\mu_0\sigma}$$

and σ is material conductivity.

The slot top contributes an additional reactive impedance:

$$Z_t = j\omega\mu_0 \frac{d}{u}$$

where d is the radial height of the slot top (depression) and u is the top width.

Total impedance is $Z_s = Z_b + Z_t$. This is plotted in magnitude-angle format in Figure 7 and in real and reactive form in Figure 8. For reference and for 'idiot check', the values of resistance and reactance are included as dotted lines. As expected, the low frequency limits of both resistance and reactance match. Note the actual reactance is always a bit below the limit, and the actual resistance is always above the limit.

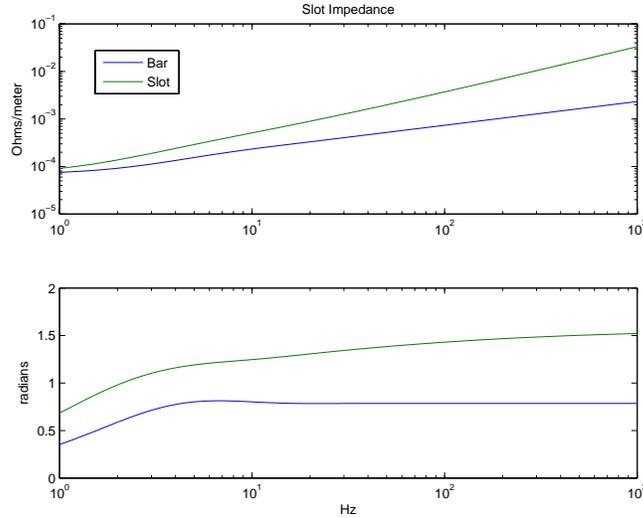


Figure 7: Slot Impedance: Magnitude-angle

Finally, we insert this into the machine model of Problem 1. To do this, we first of all compute the resistance and reactance of the slot per unit length if it were not affected by diffusion:

$$R_{20} = \frac{1}{wh\sigma}$$

$$X_{20} = \omega\mu_0 \left(\frac{h_d}{w_d} + \frac{1}{3} \frac{h}{w} \right)$$

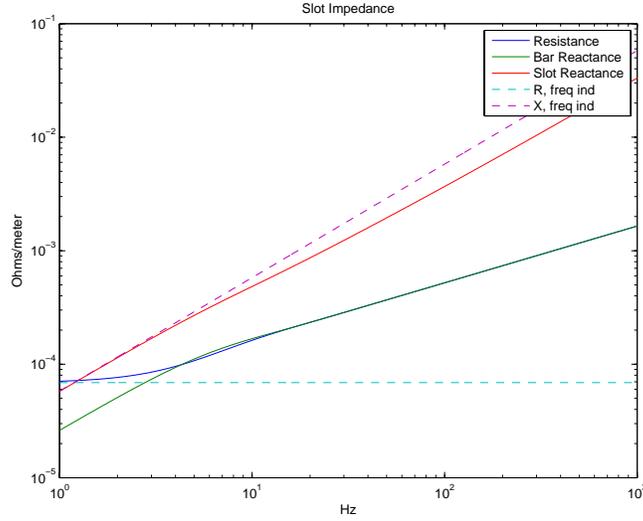


Figure 8: Slot Impedance: Real and Reactive

Then, using the computation for slot impedance per unit length, and calling that Z_s , and using R_{2z} and X_{2z} as the base values used in Problem 1, we have:

$$R_2 = \operatorname{Re}\{Z_s\} \frac{R_{2z}}{R_{20}}$$

$$X_2 = 0.2X_{2z} + 0.8\operatorname{Im}\{Z_s\} \frac{X_{2z}}{sX_{20}}$$

The second of the two expressions relates to the fact that reactance on the armature side of the air-gap is $1/s$ times the imaginary part of the imaginary part of slot impedance.

The resulting torque-speed curve is compared with the frequency independent parameter torque-speed curve in Figure 9. Note that this shows an effect far more dramatic than one would expect in a real machine (5 cm is a really deep bar). The peak torque is impacted both by an increase in resistance and by a reduction in reactance.

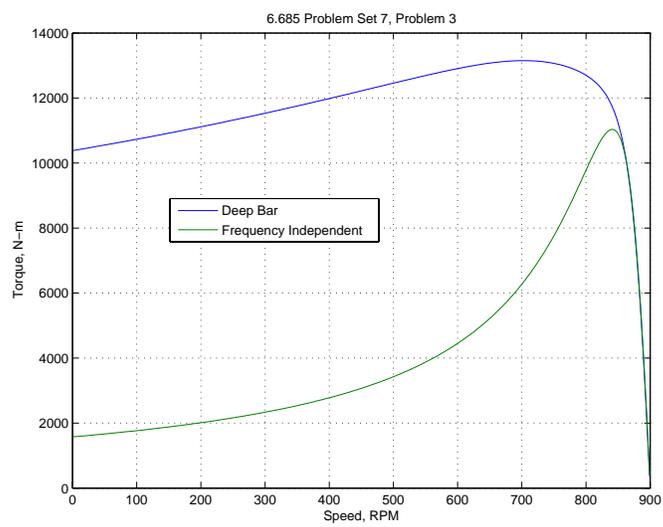


Figure 9: Torque-Speed Curve with Deep Rotor Bars

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% 6.685 Problem Set 7, Problem 1 (2013)
% this is a 450 kW induction motor

V = 600/sqrt(3);      % line-neutral voltage (RMS, line-neutral)
f = 60;              % Line frequency
p = 4;              % number of pole pairs
x1 = .038;          % stator leakage reactance
x2 = .114;          % rotor leakage reactance
r1 = .017;          % stator resistance
r2 = .010;          % rotor resistance
xm = 3.5;           % magnetizing reactance

Pc0 = 10000;        % core base loss (w)
slc = .025;         % stray load coefficient
epsf = 1.8;         % core loss frequency exponent
epsb = 2.2;         % core loss flux exponent
Rc = 3*V^2/Pc0;     % core parallel element

% part 1: ordinary torque/speed curve
% use the parallel core loss element
om = 2*pi*f;        % frequency in radians/second
s = logspace(-3,0,500); % use this range of slip

Zr = j*x2 + r2 ./ s; % rotor impedance
Zm = j*xm*Rc/(j*xm + Rc); % magnetizing element impedance
Zag = Zr .* Zm ./ (Zr + Zm); % air-gap impedance
Zt = j*x1 + r1 + Zag; % terminal impedance

it = V ./ Zt;      % terminal current
i2 = it .* Zm ./ (Zm + Zr); % rotor current
Pag = 3 .* abs(i2) .^2 .* r2 ./ s; % air-gap power
T = (p/om) .* Pag; % this is torque
omm = (om/p) .* (1 - s); % mechanical speed
N = (60/(2*pi)) .* omm; % in RPM, for convenience
Pm = Pag .* (1 - s) .* (1 - slc); % mechanical power out
Pin = 3 .* real(V .* conj(it)); % real power in
Pa = 3 * V .* abs(it); % apparent power in

figure(1)
clf
plot(N, T)
title('6.685 Problem Set 7, Problem 1')
ylabel('Torque, N-m')
xlabel('Speed, RPM')
grid on

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```

% save for later
Tz = T;

figure(2)
clf
plot(N, Pm)
title('6.685 Problem Set 7, Problem 1')
ylabel('Output Power, W')
xlabel('Speed, RPM')
grid on

% now we should find the rough range of slips for
% the specified range of power

[imaxp, imaxp] = max(Pm);           % index of maximum power

Pmin = 100000;                       % minimum power of range
Pmax = 400000;                       % maximum power of range
for k = 1:imaxp
    if Pm(k) < Pmin & Pm(k+1) > Pmin
        ismin = k;
    end
    if Pm(k) < Pmax & Pm(k+1) > Pmax
        ismax = k;
    end
end

Pmr = Pm(ismin:ismax);               % over this range of output
Pinr = Pin(ismin:ismax);             % input power for this range
Par = Pa(ismin:ismax);               % apparent power in

eff = Pmr ./ Pinr;                   % efficiency
pf = Pinr ./ Par;                    % power factor

figure(3)
clf
plot(Pmr, eff, Pmr, pf)
title('6.685 Problem Set 7, Problem 1')
ylabel('Per Unit')
xlabel('Power, watts')
grid on
legend('eff', 'pf')

% now on to problem 2

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```

% shameless hack to get parameters right
omz = om;
x1z = x1;
x2z = x2;
xmz = xm;
Rcz = Rc;
fz = f;

freq = [20 40 60 80 100 120];
volts = (600/sqrt(3)) .* [20/60 40/60 1 1 1 1];

figure(4)
clf
hold on

for k = 1:length(freq)
    V = volts(k);
    om = 2*pi*freq(k);
    x1 = x1z *om/omz;           % correct reactances
    x2 = x2z * om/omz;
    xm = xmz * om/omz;
    Rc = Rcz*(fz/f)^.2;       % and core for frequency

    Zr = j*x2 + r2 ./ s;      % rotor impedance
    Zm = j*xm*Rc/(j*xm + Rc); % magnetizing element impedance
    Zag = Zr .* Zm ./ (Zr + Zm); % air-gap impedance
    Zt = j*x1 + r1 + Zag;     % terminal impedance

    it = V ./ Zt;           % terminal current
    i2 = it .* Zm ./ (Zm + Zr); % rotor current
    Pag = 3 .* abs(i2) .^2 .* r2 ./ s; % air-gap power
    T = (p/om) .* Pag;     % this is torque
    omm = (om/p) .* (1 - s); % mechanical speed
    N = (60/(2*pi)) .* omm; % in RPM, for convenience

    plot(N, T)
end

hold off
title('6.685 Problem Set 7, Problem 2')
ylabel('Torque, N-m')
xlabel('Speed, RPM')
grid on

% ok: now we are going to look for a single power over a range

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% of speeds.

freq = 27:1:125;

eff = zeros(size(freq));
pf = zeros(size(freq));
Nd = zeros(size(freq));

for k = 1:length(freq)
    if freq(k) < 60
        V = (600/sqrt(3)) * freq(k)/60;
    else
        V = 600/sqrt(3);
    end
    om = 2*pi*freq(k);
    x1 = x1z * om/omz;
    x2 = x2z * om/omz;
    xm = xmz * om/omz;
    Rc = RcZ * (f/fz)^.2;

    Zr = j*x2 + r2 ./ s;           % rotor impedance
    Zm = j*xm*Rc/(j*xm + Rc);     % magnetizing element impedance
    Zag = Zr .* Zm ./ (Zr + Zm);  % air-gap impedance
    Zt = j*x1 + r1 + Zag;        % terminal impedance

    it = V ./ Zt;                % terminal current
    i2 = it .* Zm ./ (Zm + Zr);   % rotor current
    Pag = 3 .* abs(i2) .^2 .* r2 ./ s; % air-gap power
    T = (p/om) .* Pag;           % this is torque
    omm = (om/p) .* (1 - s);      % mechanical speed
    N = (60/(2*pi)) .* omm;      % in RPM, for convenience
    Pm = Pag .* (1 - s) .* (1 - slc); % mechanical power out
    Pin = 3 .* real(V .* conj(it)); % real power in
    Pa = 3 * V .* abs(it);       % apparent power in

    % now to find the specific operating point
    [maxp, imaxp] = max(Pm);      % index of maximum power
    for kk = 1:imaxp
        if Pm(kk) < 175000 & Pm(kk+1)>175000
            sr = s(kk) + (s(kk+1)-s(kk)) * (37300-Pm(kk))/(Pm(kk+1)-Pm(kk));
            break
        end
    end
end
% now we have the operating point, away we go
% for operation at that value of slip:

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Zr = j*x2 + r2 / sr;           % rotor impedance
Zm = j*xm*Rc/(j*xm + Rc);     % magnetizing element impedance
Zag = Zr * Zm / (Zr + Zm);    % air-gap impedance
Zt = j*x1 + r1 + Zag;         % terminal impedance

it = V / Zt;                  % terminal current
i2 = it * Zm / (Zm + Zr);     % rotor current
Pag = 3 * abs(i2) ^2 * r2 / sr; % air-gap power
T = (p/om) * Pag;             % this is torque
omm = (om/p) * (1 - sr);      % mechanical speed
Nd(k) = (60/(2*pi)) * omm;    % in RPM, for convenience
Pm = Pag * (1 - sr) * (1 - slc); % mechanical power out
Pin = 3 * real(V * conj(it)); % real power in
Pa = 3 * V * abs(it);         % apparent power in
eff(k) = Pm/Pin;
pf(k) = Pin/Pa;
end

figure(5)
clf
plot(Nd, eff, Nd, pf)
title('6.685 Problem Set 7, Problem 2 175 kW')
ylabel('Per Unit')
xlabel('Speed, RPM')
grid on
legend('eff', 'pf')

% part 3: ordinary torque/speed curve with deep bar
% use the parallel core loss element
w = .005;           % slot width
h = .05;           % slot height
wd = .0005;        % depression width
hd = .002;         % depression height
sig = 5.81e7;      % IASC
muzero = pi*4e-7;
om = 2*pi*f;       % frequency in radians/second
s = logspace(-3,0,500); % use this range of slip

x2r = .2*x2z;      % 'remainder' part of x2
x2n = .8*x2z;     % normalize part of x2
r2n = r2;
xm = xmz;
V=600/sqrt(3);

```

```

% get low frequency part of slot impedance
r20 = 1/(sig*h*w); % resistance
x20 = s .* om*muzero*(hd/wd+h/(3*w)); % reactance
% now to get the slot parameters
delt = sqrt(2 ./ (s .* om*sig*muzero)); % skin depth
z_s = ((1+j) ./ (sig*w .*delt)) .* coth(h*(1+j) ./ delt);
z_t = z_s + j*muzero*(hd/wd)*om .* s;

x2 = x2r + imag(z_t) .* x2n/x20;
r2 = real(z_t) .* r2n/r20; % rotor resistance

Zr = j*x2 + r2 ./ s; % rotor impedance
Zm = j*xm*Rc/(j*xm + Rc); % magnetizing element impedance
Zag = Zr .* Zm ./ (Zr + Zm); % air-gap impedance
Zt = j*x1z + r1 + Zag; % terminal impedance

it = V ./ Zt; % terminal current
i2 = it .* Zm ./ (Zm + Zr); % rotor current
Pag = 3 .* abs(i2) .^2 .* r2 ./ s;% air-gap power
T = (p/om) .* Pag; % this is torque
omm = (om/p) .* (1 - s); % mechanical speed
N = (60/(2*pi)) .* omm; % in RPM, for convenience

figure(6)
clf
plot(N, T, N, Tz)
title('6.685 Problem Set 7, Problem 3')
ylabel('Torque, N-m')
xlabel('Speed, RPM')
legend('Deep Bar', 'Frequency Independent')
grid on

```

```

% Problem Set 7, Problem 3: deep bar

muzero = pi*4e-7;           % muzero
sig = 5.81e7;              % IASC
f = logspace(0,3,1000);    % range of frequencies
om = 2*pi .* f;           % in radians/second
w = .005;                  % slot width
h = .05;                  % slot height
wd = .0005;                % depression width
hd = .002;                % depression height
delt = sqrt(2 ./ (om .* sig*muzero)); % skin depth
z_s = (((1+j)/(sig*w)) ./delt) .* coth(h*(1+j) ./ delt);
z_t = z_s + j*muzero*hd/wd .* om;

figure(1)
subplot 211
loglog(f, abs(z_s), f, abs(z_t))
title('Slot Impedance')
ylabel('Ohms/meter')
subplot 212
semilogx(f, angle(z_s), f, angle(z_t))
ylabel('radians')
xlabel('Hz')
legend('Bar', 'Slot')

x_f = om .* muzero*(hd/wd + h/(3*w));
r_f = ones(size(f)) ./ (sig*h*w);
figure(2)
loglog(f, real(z_s), f, imag(z_s), f, imag(z_t),f,r_f,'--', f, x_f,'--')
title('Slot Impedance')
ylabel('Ohms/meter')
xlabel('Hz')
legend('Resistance', 'Bar Reactance', 'Slot Reactance', 'R, freq ind', 'X, freq ind')

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