Massachusetts Institute of Technology
**Department of Electrical Engineering and Computer Science**
6.685 Electric Machines

Problem Set 10 — Issued November 11, 2013
Due November 20, 2013

**Problem 1:** Permanent Magnets

This problem is related to permanent magnet motors. We are going to obtain field patterns as you might measure them with a flux meter. This problem should be worked in cylindrical coordinates. As it is a permanent magnet problem, we can find magnetic field as the gradient of a scalar potential:

$$\vec{H} = -\nabla\psi$$

In cylindrical coordinates, the solutions for the potential are polynomials:

$$\psi = Ar^{\pm p}\cos p\theta \qquad or\, Ar^{\pm p}\sin p\theta$$

You might recall working similar problems in rectangular coordinates, in which the solutions to similar problems are growing and decaying exponential functions.

Note that most problems will involve an annular space, with boundary conditions at an inner and outer radius. If there is no inner radius boundary condition (the center is in the region of interest, the solution with a negative exponent must have an amplitude of zero, so the potential does not 'blow up'. If there is no outer radius (the region goes to $\infty$), the solution with a positive exponent must have zero amplitude.

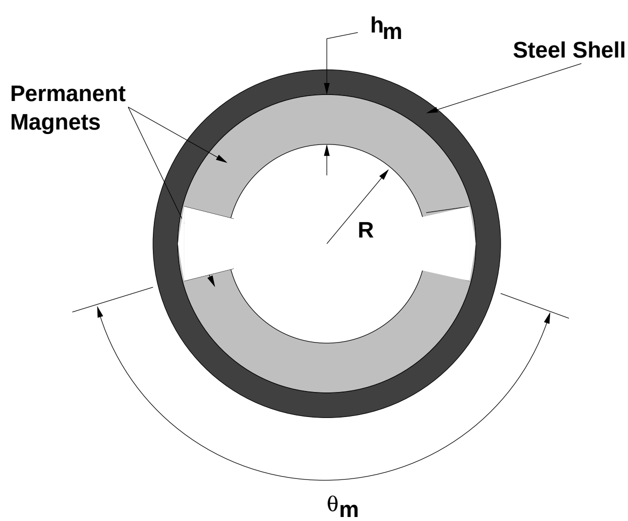


Figure 1: Permanent Magnet Stator

Figure 1 shows a two-pole ($p = 1$) permanent magnet stator. Two permanent magnets are mounted on the inside of a steel shell that serves as both structure and magnetic return path.

This could be a part of a permanent magnet DC motor, with a commutator. Or it could be the rotor of an inside-out structure such as is used in small fan motors. Dimensions are:

| | | | |
|---|---|---|---|
| Magnet inside radius | R | 4 | cm |
| Magnet height | $h_m$ | 2.5 | mm |
| Magnet angular width | $\theta_m$ | $\frac{5\pi}{6}$ | 150° |

Assume the remanent flux density of the magnets is $B_r = 0.4$ T. This problem is meant to be worked using a Fourier Series in the $\theta$- direction. Be sure to use enough space harmonics to get a good representation of the actual fields.

1. To start, assume that the problem is as you see it: there is no rotor so the region at radius less than $R$ is empty. Plot radial and azimuthal field as a function of $\theta$ at the inside radius of the magnets ($R$) and at a radius $R - 1$mm.
2. Next, assume that there is a rotor, which for our purposes can be considered to be a ferromagnetic cylinder with a radius of $R_i = R - g$, with a gap dimension of $g = 1.0$ mm. Plot radial and azimuthal field at the radius of the inner surface of the magnets ($R$) and radial field at the surface of the rotor $R_i$. (Of course, azimuthal field at that radius is not very interesting, right?)

**Proglem 2:** Induction Motor Simulation

The objective of this problem is to see how reduced order models of electric machines can be used to give approximate results and to give some sense of how those approximations miss certain features of machine operation.

A large induction motor intended to drive a fan can be represented by the simple equivalent circuit with the following parameters:

| | | | |
|---|---|---|---|
| Stator Resistance | $R_1$ | 0.460 | Ω |
| Rotor Resistance | $R_2$ | 0.433 | Ω |
| Stator Leakage Reactance | $X_1$ | 3.51 | Ω |
| Rotor Leakage Reactance | $X_2$ | 5.05 | Ω |
| Magnetizing Reactance | $X_m$ | 95.6 | Ω |

This motor is subjected to an across-the-line start, and in this problem set we will simulate that start. For each part,

1. The machine and load inertia is equal to $80kg - m^2$, and
2. The machine is driving a fan load. For the purpose of this problem, assume that power drawn by the fan is exactly a cubic function of speed, so that load torque is proportional to speed squared. Assume that the fan load would be equal to 800 kW at synchronous speed (which you will not, of course, quite achieve).

The fan is operated by a voltage source that is 60 Hz, 6,000 volts, RMS, line-line. (Be careful to get phase voltage right here!). It is an 8-pole machine so its synchronous speed is 900 RPM.

For each part of the problem, calculate and plot:

1. Speed (RPM) vs. time (simulate for 5 seconds).
2. Real power drawn from the source over the same time.

The three cases to simulate are really three different and progressively more detailed models of the machine. These are:

1. A 'First Order' model which assumes that the stator and rotor are both in *electical* steady state so that the only dynamic (state) variable is rotor speed.
2. A 'Third Order' model which uses the rotor variables ($\psi_{dr}$ and $\psi_{qr}$ and, of course, rotor speed but which assumes that stator variables can be either neglected or assumed to be in steady state conditions. You can also ignore stator resistance in calculating the stator quantities.
3. A 'Fifth Order' model in which both stator and rotor variables are important.

Of course we are ignoring the possibility of deep bar (diffusion) effects here, so these simulations may not be all that realistic, but they do have some interesting features. Try to plot all three sets of plots on equivalent sets of axes so that the important features of each can be seen.

**Problem 3:** Doubly Fed Induction Generator

This is about a three-phase wound-rotor induction generator that might be used as a wind turbine generator. The stator and rotor windings are identical, except for the numbers of turns. It has characteristics as shown here:

| | | |
|---|---|---|
| Number of Poles | 2p | 6 |
| Armature Phase Self Inductance | $L_a$ | 3.5 mHy |
| Armature Phase-to-Phase Mutual Inductance | $L_{ab}$ | -1.75 mHy |
| Rotor Phase Self Inductance | $L_A$ | 31.5 mHy |
| Rotor Phase-to-Phase Mutual Inductance | $L_{AB}$ | -15.75 mHy |
| Rotor to Stator (Peak) Mutual Inductance | $L_{aA}$ | 10.37 mHy |
| Effective Transformer Turns Ratio | $\frac{N_r}{N_s}$ | 3 |
| Nominal Rotational Speed | | 1200 RPM |
| Terminal Voltage (RMS, Line-Line) | $V_a$ | 690 v |
| Rated Power | | 2,400 kVA |
| Frequency | | 60 Hz |

The rotor windings are connected to a set of slip rings and so can be driven by an inverter as shown in Figure 2. The inverter is part of a bidirectional AC/DC/AC converter with the other end connected directly to the power system. Assume that the 'line side' converter interacts with the machine stator (and power bus) terminals at unity power factor (that is, the reactive power either drawn or supplied by the right-hand end of the converter is zero).

Assume that the load is drawing P=2,000 kW, Q=500 kVAR. Ignoring losses in the system, find and plot the following quantities over a speed range of between 70% and 130% of synchronous:

1. Real and Reactive Power *out of* the stator winding
2. Real and Reactive Power *in to* the slip rings (and rotor winding)
3. Power delivered by the wind turbine

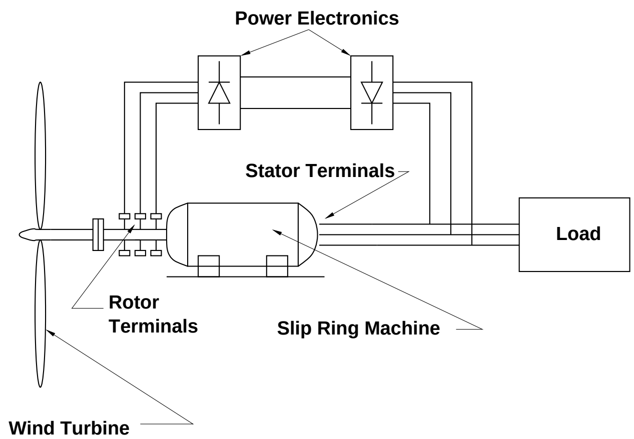


Figure 2: Wind Turbine Generator Setup

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