Derivative of $\cos x$.

What is the specific formula for the derivative of the function $\cos x$?

This calculation is very similar to that of the derivative of $\sin(x)$. If you get stuck on a step here it may help to go back and review the corresponding step there.

As in the calculation of $\frac{d}{dx}\sin x$, we begin with the definition of the derivative:

$$\frac{d}{dx}\cos x = \lim_{\Delta x\to 0}\frac{\cos(x+\Delta x)-\cos(x)}{\Delta x}$$

Use the angle sum formula $\cos(a+b)=\cos(a)\cos(b)-\sin(a)\sin(b)$ and then simplify:

$$\begin{aligned}
\frac{d}{dx}\cos x &= \lim_{\Delta x\to 0}\frac{\cos x\cos\Delta x-\sin x\sin\Delta x-\cos x}{\Delta x}\\
&= \lim_{\Delta x\to 0}\left[\frac{\cos x\cos\Delta x-\cos x}{\Delta x}+\frac{-\sin x\sin\Delta x}{\Delta x}\right]\\
&= \lim_{\Delta x\to 0}\left[\frac{\cos x(\cos\Delta x-1)}{\Delta x}+\frac{-\sin x\sin\Delta x}{\Delta x}\right]\\
&= \lim_{\Delta x\to 0}\left[\cos x\left(\frac{\cos\Delta x-1}{\Delta x}\right)+(-\sin x)\left(\frac{\sin\Delta x}{\Delta x}\right)\right]\\
\frac{d}{dx}\cos x &= \lim_{\Delta x\to 0}\cos x\left(\frac{\cos\Delta x-1}{\Delta x}\right)+\lim_{\Delta x\to 0}(-\sin x)\left(\frac{\sin\Delta x}{\Delta x}\right)
\end{aligned}$$

Once again we use the following (unproven) facts:

$$\begin{aligned}
\lim_{\Delta x\to 0}\frac{\cos\Delta x-1}{\Delta x} &= 0 \text{ (A)}\\
\lim_{\Delta x\to 0}\frac{\sin\Delta x}{\Delta x} &= 1. \text{ (B)}
\end{aligned}$$

And we conclude:

$$\begin{aligned}
\frac{d}{dx}\cos x &= \lim_{\Delta x\to 0}\cos x\left(\frac{\cos\Delta x-1}{\Delta x}\right)+\lim_{\Delta x\to 0}(-\sin x)\left(\frac{\sin\Delta x}{\Delta x}\right)\\
&= \cos x\cdot 0+(-\sin x)\cdot 1\\
\frac{d}{dx}\cos x &= -sin(x).
\end{aligned}$$