

The Indeterminate Form 0^0

We next consider the limit:

$$\lim_{x \rightarrow 0^+} x^x.$$

Can we compute this?

There are many different indeterminate forms; x^x is one of the simpler examples. In this case, because x is a moving exponent, we can use a trick to evaluate the limit.

Since we have a moving exponent, we will use base e . We rewrite our original expression as follows:

$$x^x = e^{x \ln x}.$$

Now we can focus our attention on the exponent:

$$\begin{aligned} \lim_{x \rightarrow 0^+} x \ln x &= \lim_{x \rightarrow 0^+} \frac{\ln x}{1/x} \\ &= \lim_{x \rightarrow 0^+} \frac{1/x}{-1/x^2} \quad (\text{l'Hop}) \\ &= \lim_{x \rightarrow 0^+} -x \\ &= 0. \end{aligned}$$

Therefore,

$$\begin{aligned} \lim_{x \rightarrow 0^+} x^x &= \lim_{x \rightarrow 0^+} e^{x \ln x} \\ &= e^0 \\ &= 1. \end{aligned}$$

This was relatively easy to calculate because we have so many powerful tools to work with.

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