

$$\lim_{x \rightarrow \infty} (x^{1/x})$$

Use an extension of l'Hôpital's rule to compute $\lim_{x \rightarrow \infty} (x^{1/x})$.

Solution

This calculation is very similar to the calculation of $\lim_{x \rightarrow 0^+} x^x$ presented in lecture, except that instead of the indeterminate form 0^0 we instead have ∞^0 .

As before, we use the exponential and natural log functions to rephrase the problem:

$$x^{1/x} = e^{\ln x^{1/x}} = e^{\frac{\ln x}{x}}.$$

Thus, $\lim_{x \rightarrow \infty} x^{1/x} = \lim_{x \rightarrow \infty} e^{\frac{\ln x}{x}}$. Since the function e^t is continuous,

$$\lim_{x \rightarrow \infty} e^{\frac{\ln x}{x}} = e^{\lim_{x \rightarrow \infty} \frac{\ln x}{x}}.$$

We can now focus our attention on the limit in the exponent; $\lim_{x \rightarrow \infty} \frac{\ln x}{x}$ is in the indeterminate form $\frac{\infty}{\infty}$, so l'Hôpital's rule is applicable.

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{\ln x}{x} &= \lim_{x \rightarrow \infty} \frac{1/x}{1} && \text{(provided the limit exists)} \\ &= \frac{0}{1} \\ &= 0 \end{aligned}$$

We conclude that $\lim_{x \rightarrow \infty} x^{1/x} = e^{\lim_{x \rightarrow \infty} \frac{\ln x}{x}} = 1$.

This implies that the n^{th} root of n approaches 1 as n approaches infinity.

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