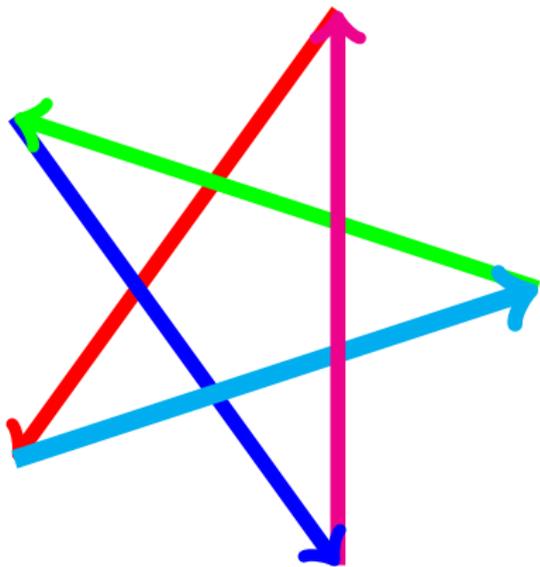


Studio 2
18.05 Spring 2014



Expected Value

If X is a random variable that takes values x_1, x_2, \dots, x_n then the *expected value* of X is defined by

$$E(X) = p(x_1)x_1 + p(x_2)x_2 + \dots + p(x_n)x_n = \sum_{i=1}^n p(x_i) x_i$$

- Weighted average
- Measure of central tendency

Properties of $E(X)$

1. $E(X + Y) = E(X) + E(Y)$
2. $E(aX + b) = aE(X) + b$
3. $E(h(X)) = \sum_i h(x_i) p(x_i)$

Examples

Example 1. Find $E(X)$

1. $X:$ 3 4 5 6

2. pmf: 1/4 1/2 1/8 1/8

3. $E(X) = 3/4 + 4/2 + 5/8 + 6/8 = 33/8$

Example 2. Suppose $X \sim \text{Bernoulli}(p)$. Find $E(X)$.

1. $X:$ 0 1

2. pmf: 1 - p p

3. $E(X) = (1 - p) \cdot 0 + p \cdot 1 = p.$

Example 3. Suppose $X \sim \text{Binomial}(12, .25)$. Find $E(X)$.

$X = X_1 + X_2 + \dots + X_{12}$, where $X_i \sim \text{Bernoulli}(.25)$. Therefore

$$E(X) = E(X_1) + E(X_2) + \dots + E(X_{12}) = 12 \cdot (.25) = 3$$

In general if $X \sim \text{Binomial}(n, p)$ then $E(X) = np$.

Board Question

Suppose (hypothetically!) that everyone at your table gets up, does a board question, and sits back down at random (i.e., all seating arrangements are equally likely).

What is the expected number of people who return to their original seat?

R Exercises

Suppose $Y \sim \text{Binomial}(8, .6)$.

1. Run a simulation with 1000 trials to estimate $P(Y = 6)$ and $P(Y \leq 6)$
2. Use R and the formula for binomial probabilities to compute $P(Y=6)$ exactly.

R Exercises

3. A friend has a coin with probability .6 of heads. She proposes the following gambling game.

- You will toss it 10 times and count the number of heads.
- The amount you win or lose on k heads is given by $k^2 - 7k$

(a) Plot the payoff function.

(b) Make an exact computation using R to decide if this is a good bet.

(c) Run a simulation and see that it approximates your computation in part (b).

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