Problem Set 4

AG §2.3, pp 80–82: 2, 3, 8, 10, 11, 15 (in 15b assume $c_i > 0$).

AG §2.4 pp 88–89: 2, 3, 6, 7.

AG §1.3, pp 39–42: 19, 20.

The property (†) was introduced by Caratheodory and gives a slick approach to defining measurable sets. The idea of (†) is that if we formulate the correct notion of inner measure, then measurable sets should be the ones for which the inner and outer measures of a set coincide. The inner measure is expressed using outer measure of the complementary set. The idea is analogous to upper and lower Riemann sums.

The subtlety is that the condition in (†) is imposed not just on A and A^c, but also on each $A \cap E$ and $A^c \cap E$ for every set E. Note that E is permitted to be any set, not just a measurable set. If (†) is to substitute as a definition of measurable set, it is incoherent to insist that E be measurable — we don't yet know what a measurable set is!

18.103 Fourier Analysis
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