Problem Set 9

Do AG §3.5/ 3, 4, 7, 8, and the following additional problem.

(Alternative proof of Fourier inversion on $\mathbb{R}$ using Fourier inversion on $\mathbb{R}/2L\mathbb{Z}$.)

a) For $f \in C^\infty(\mathbb{R})$ periodic of period $2L$ define

$$c_n = \frac{1}{2L}\int_{-L}^{L} f(x)e^{-in\pi x/L}dx$$

Show (by change of variables) that

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{i\pi nx/L}$$

b) For $g \in C_0^\infty(\mathbb{R})$, i. e., g infinitely differentiable with compact support, define

$$\hat{g}(\xi) = \int_{-\infty}^{\infty} g(x)e^{-ix\xi}dx$$

Use part (a) and justify the passage to the limit as $L \to \infty$ to prove that

$$g(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{g}(\xi)e^{ix\xi}d\xi$$

(You may use the fact that $\hat{g} \in \mathcal{S}$, the Schwartz class.)