

18.443 Problem Set 10 Spring 2015
Statistics for Applications
Due Date: 5/15/2015
prior to 3:00pm

Problems from John A. Rice, Third Edition. [*Chapter.Section.Problem*]

1. 12.5.1

Solution: See R script/html *Problem_12.5.1.r/html*

2. 12.5.6

Prove this version of the Bonferroni inequality:

$$P(\cap_{i=1}^n A_i) \geq 1 - \sum_{i=1}^n P(A_i^c)$$

Let $A_* = \cap_{i=1}^n A_i$. Then

$$A_*^c = \cup_{k=1}^n A_k^c$$

So $P(A_*^c) = P(\cup_{k=1}^n A_k^c) \leq \sum_{k=1}^n P(A_k^c)$

Because $P(A_*) = 1 - P(A_*^c)$, it follows that

$$P(A_*) \geq 1 - \sum_{k=1}^n P(A_k^c)$$

In the context of simultaneous confidence intervals

A_i is the the event that the i th confidence interval contains the associated parameter

$\cap_{i=1}^n A_i$ is the the event that all confidence intervals contains the associated parameters simultaneously.

3. 12.5.17. Find the mle's of the parameters $\alpha_i, \beta_j, \delta_{ij}$, and μ of the model for the two-way layout.

The model for the two-way layout is given by:

$$Y_{ijk} = \mu + \alpha_i + \beta_j + \delta_{ij} + \epsilon_{ijk}$$

where

$$\sum_{i=1}^I \alpha_i = 0$$

$$\sum_{j=1}^J \beta_j = 0$$

$$\sum_{i=1}^I \delta_{ij} = \sum_{j=1}^J \delta_{ij} = 0$$

As noted in Rice (p. 493) the log likelihood is

$$l = -\frac{IJK}{2} \log(2\pi\sigma^2) - \frac{1}{2\sigma^2} \sum_{i=1}^I \sum_{j=1}^J \sum_{k=1}^K (Y_{ijk} - \mu - \alpha_i - \beta_j - \delta_{ij})^2$$

Solving the following equations:

$$\frac{\partial l}{\partial \mu} = 0 \implies \hat{\mu} = \overline{Y_{...}}$$

$$\frac{\partial l}{\partial \alpha_i} = 0 \implies \hat{\alpha}_i = \overline{Y_{i..}} - \hat{\mu}$$

$$\frac{\partial l}{\partial \beta_j} = 0 \implies \hat{\beta}_j = \overline{Y_{.j.}} - \hat{\mu}$$

$$\frac{\partial l}{\partial \delta_{ij}} = 0 \implies \hat{\delta}_{ij} = \overline{Y_{ij.}} - \hat{\mu} - \hat{\alpha}_i - \hat{\beta}_j$$

These terms yield the answer formulas of the problem.

4. 13.8.1. See R script html [Problem_13_8_1.html](#)
5. 13.8.25. See R script html [Problem_13_8_25.html](#)

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