18.600: Lecture 1

Permutations and combinations, Pascal's triangle, learning to count

Scott Sheffield

MIT

Outline

Outline

Politics

- Suppose that, in some election, betting markets place the probability that your favorite candidate will be elected at 58 percent. Price of a contact that pays 100 dollars if your candidate wins is 58 dollars.

Politics

- Suppose that, in some election, betting markets place the probability that your favorite candidate will be elected at 58 percent. Price of a contact that pays 100 dollars if your candidate wins is 58 dollars.
- Market seems to say that your candidate will probably win, if "probably" means with probability greater than .5.

Politics

- Suppose that, in some election, betting markets place the probability that your favorite candidate will be elected at 58 percent. Price of a contact that pays 100 dollars if your candidate wins is 58 dollars.
- Market seems to say that your candidate will probably win, if "probably" means with probability greater than .5.
- The price of such a contract may fluctuate in time.

Politics

- Suppose that, in some election, betting markets place the probability that your favorite candidate will be elected at 58 percent. Price of a contact that pays 100 dollars if your candidate wins is 58 dollars.
- Market seems to say that your candidate will probably win, if "probably" means with probability greater than .5.
- The price of such a contract may fluctuate in time.
- Let $X(t)$ denote the price at time t.

Politics

- Suppose that, in some election, betting markets place the probability that your favorite candidate will be elected at 58 percent. Price of a contact that pays 100 dollars if your candidate wins is 58 dollars.
- Market seems to say that your candidate will probably win, if "probably" means with probability greater than .5.
- The price of such a contract may fluctuate in time.
- Let $X(t)$ denote the price at time t.
- Suppose $X(t)$ is known to vary continuously in time. What is probability p it reaches 59 before 57?

Politics

- Suppose that, in some election, betting markets place the probability that your favorite candidate will be elected at 58 percent. Price of a contact that pays 100 dollars if your candidate wins is 58 dollars.
- Market seems to say that your candidate will probably win, if "probably" means with probability greater than .5.
- The price of such a contract may fluctuate in time.
- Let $X(t)$ denote the price at time t.
- Suppose $X(t)$ is known to vary continuously in time. What is probability p it reaches 59 before 57?
- If $p > .5$, we can make money in expecation by buying at 58 and selling when price hits 57 or 59.

Politics

- Suppose that, in some election, betting markets place the probability that your favorite candidate will be elected at 58 percent. Price of a contact that pays 100 dollars if your candidate wins is 58 dollars.
- Market seems to say that your candidate will probably win, if "probably" means with probability greater than .5.
- The price of such a contract may fluctuate in time.
- Let $X(t)$ denote the price at time t.
- Suppose $X(t)$ is known to vary continuously in time. What is probability p it reaches 59 before 57?
- If $p > .5$, we can make money in expecation by buying at 58 and selling when price hits 57 or 59.
- If $p < .5$, we can sell at 58 and buy when price hits 57 or 59.

Politics

- Suppose that, in some election, betting markets place the probability that your favorite candidate will be elected at 58 percent. Price of a contact that pays 100 dollars if your candidate wins is 58 dollars.
- Market seems to say that your candidate will probably win, if "probably" means with probability greater than .5.
- The price of such a contract may fluctuate in time.
- Let $X(t)$ denote the price at time t.
- Suppose $X(t)$ is known to vary continuously in time. What is probability p it reaches 59 before 57?
- If $p > .5$, we can make money in expecation by buying at 58 and selling when price hits 57 or 59.
- If $p < .5$, we can sell at 58 and buy when price hits 57 or 59.
- Efficient market hypothesis (a.k.a. "no free money just lying around" hypothesis) suggests $p = .5$ (with some caveats...)

Politics

- Suppose that, in some election, betting markets place the probability that your favorite candidate will be elected at 58 percent. Price of a contact that pays 100 dollars if your candidate wins is 58 dollars.
- Market seems to say that your candidate will probably win, if "probably" means with probability greater than .5.
- The price of such a contract may fluctuate in time.
- Let $X(t)$ denote the price at time t.
- Suppose $X(t)$ is known to vary continuously in time. What is probability p it reaches 59 before 57?
- If $p > .5$, we can make money in expecation by buying at 58 and selling when price hits 57 or 59.
- If $p < .5$, we can sell at 58 and buy when price hits 57 or 59.
- Efficient market hypothesis (a.k.a. "no free money just lying around" hypothesis) suggests $p = .5$ (with some caveats...)
- Natural model for prices: repeatedly toss coin, adding 1 for heads and -1 for tails, until price hits 0 or 100.

Which of these statements is “probably” true?

- 1. $X(t)$ will go below 50 at some future point.

Which of these statements is “probably” true?

- 1. $X(t)$ will go below 50 at some future point.
- 2. $X(t)$ will get all the way below 20 at some point

Which of these statements is "probably" true?

- 1. $X(t)$ will go below 50 at some future point.
- 2. $X(t)$ will get all the way below 20 at some point
- 3. $X(t)$ will reach both 70 and 30, at different future times.

Which of these statements is “probably” true?

- 1. $X(t)$ will go below 50 at some future point.
- 2. $X(t)$ will get all the way below 20 at some point
- 3. $X(t)$ will reach both 70 and 30, at different future times.
- 4. $X(t)$ will reach both 65 and 35 at different future times.

Which of these statements is "probably" true?

- 1. $X(t)$ will go below 50 at some future point.
- 2. $X(t)$ will get all the way below 20 at some point
- 3. $X(t)$ will reach both 70 and 30, at different future times.
- 4. $X(t)$ will reach both 65 and 35 at different future times.
- 5. $X(t)$ will hit 65, then 50, then 60, then 55.

Which of these statements is "probably" true?

- 1. $X(t)$ will go below 50 at some future point.
- 2. $X(t)$ will get all the way below 20 at some point
- 3. $X(t)$ will reach both 70 and 30, at different future times.
- 4. $X(t)$ will reach both 65 and 35 at different future times.
- 5. $X(t)$ will hit 65, then 50, then 60, then 55.
- Answers: 1, 2, 4.

Which of these statements is “probably” true?

- 1. $X(t)$ will go below 50 at some future point.
- 2. $X(t)$ will get all the way below 20 at some point
- 3. $X(t)$ will reach both 70 and 30, at different future times.
- 4. $X(t)$ will reach both 65 and 35 at different future times.
- 5. $X(t)$ will hit 65, then 50, then 60, then 55.
- Answers: 1, 2, 4.
- Full explanations coming toward the end of the course.

Which of these statements is “probably” true?

- 1. $X(t)$ will go below 50 at some future point.
- 2. $X(t)$ will get all the way below 20 at some point
- 3. $X(t)$ will reach both 70 and 30, at different future times.
- 4. $X(t)$ will reach both 65 and 35 at different future times.
- 5. $X(t)$ will hit 65, then 50, then 60, then 55.
- Answers: 1, 2, 4.
- Full explanations coming toward the end of the course.
- Problem sets in this course explore applications of probability to politics, medicine, finance, economics, science, engineering, philosophy, dating, etc. Stories motivate the math and make it easier to remember.

Which of these statements is “probably” true?

- 1. $X(t)$ will go below 50 at some future point.
- 2. $X(t)$ will get all the way below 20 at some point
- 3. $X(t)$ will reach both 70 and 30, at different future times.
- 4. $X(t)$ will reach both 65 and 35 at different future times.
- 5. $X(t)$ will hit 65, then 50, then 60, then 55.
- Answers: 1, 2, 4.
- Full explanations coming toward the end of the course.
- Problem sets in this course explore applications of probability to politics, medicine, finance, economics, science, engineering, philosophy, dating, etc. Stories motivate the math and make it easier to remember.
- Provocative question: what simple advice, that would greatly benefit humanity, are we unaware of? Foods to avoid? Exercises to do? Books to read? How would we know?

Which of these statements is "probably" true?

- 1. $X(t)$ will go below 50 at some future point.
- 2. $X(t)$ will get all the way below 20 at some point
- 3. $X(t)$ will reach both 70 and 30, at different future times.
- 4. $X(t)$ will reach both 65 and 35 at different future times.
- 5. $X(t)$ will hit 65, then 50, then 60, then 55.
- Answers: 1, 2, 4.
- Full explanations coming toward the end of the course.
- Problem sets in this course explore applications of probability to politics, medicine, finance, economics, science, engineering, philosophy, dating, etc. Stories motivate the math and make it easier to remember.
- Provocative question: what simple advice, that would greatly benefit humanity, are we unaware of? Foods to avoid? Exercises to do? Books to read? How would we know?
- Let's start with easier questions.

Outline

Outline

Remark, just for fun

Permutations

Counting tricks

Binomial coefficients

Problems

Permutations

- How many ways to order 52 cards?

Permutations

- How many ways to order 52 cards?
- Answer: $52 \cdot 51 \cdot 50 \cdot \ldots \cdot 1 = 52! = 80658175170943878571660636856403766975289505600883277824 \times 10^{12}$

Permutations

- How many ways to order 52 cards?
- Answer: $52 \cdot 51 \cdot 50 \cdot \ldots \cdot 1 = 52! = 80658175170943878571660636856403766975289505600883277824 \times 10^{12}$
- n hats, n people, how many ways to assign each person a hat?

Permutations

- How many ways to order 52 cards?
- Answer: $52 \cdot 51 \cdot 50 \cdot \ldots \cdot 1 = 52! = 80658175170943878571660636856403766975289505600883277824 \times 10^{12}$
- n hats, n people, how many ways to assign each person a hat?
- Answer: $n!$

Permutations

- How many ways to order 52 cards?
- Answer: $52 \cdot 51 \cdot 50 \cdot \ldots \cdot 1 = 52! = 80658175170943878571660636856403766975289505600883277824 \times 10^{12}$
- n hats, n people, how many ways to assign each person a hat?
- Answer: $n!$
- n hats, $k < n$ people, how many ways to assign each person a hat?

Permutations

- How many ways to order 52 cards?
- Answer: $52 \cdot 51 \cdot 50 \cdot \ldots \cdot 1 = 52! = 80658175170943878571660636856403766975289505600883277824 \times 10^{12}$
- n hats, n people, how many ways to assign each person a hat?
- Answer: $n!$
- n hats, $k < n$ people, how many ways to assign each person a hat?
- $n \cdot (n-1) \cdot (n-2) \ldots (n-k+1) = n!/(n-k)!$

Permutation notation

- A **permutation** is a function from $\{1, 2, \ldots, n\}$ to $\{1, 2, \ldots, n\}$ whose range is the whole set $\{1, 2, \ldots, n\}$. If σ is a permutation then for each j between 1 and n, the the value $\sigma(j)$ is the number that j gets mapped to.

Permutation notation

- A **permutation** is a function from $\{1, 2, \ldots, n\}$ to $\{1, 2, \ldots, n\}$ whose range is the whole set $\{1, 2, \ldots, n\}$. If σ is a permutation then for each j between 1 and n, the the value $\sigma(j)$ is the number that j gets mapped to.
- For example, if $n = 3$, then σ could be a function such that $\sigma(1) = 3$, $\sigma(2) = 2$, and $\sigma(3) = 1$.

Permutation notation

- A **permutation** is a function from $\{1, 2, \ldots, n\}$ to $\{1, 2, \ldots, n\}$ whose range is the whole set $\{1, 2, \ldots, n\}$. If σ is a permutation then for each j between 1 and n, the the value $\sigma(j)$ is the number that j gets mapped to.
- For example, if $n = 3$, then σ could be a function such that $\sigma(1) = 3$, $\sigma(2) = 2$, and $\sigma(3) = 1$.
- If you have n cards with labels 1 through n and you shuffle them, then you can let $\sigma(j)$ denote the label of the card in the jth position. Thus orderings of n cards are in one-to-one correspondence with permutations of n elements.

Permutation notation

- A **permutation** is a function from $\{1, 2, \ldots, n\}$ to $\{1, 2, \ldots, n\}$ whose range is the whole set $\{1, 2, \ldots, n\}$. If σ is a permutation then for each j between 1 and n, the the value $\sigma(j)$ is the number that j gets mapped to.
- For example, if $n = 3$, then σ could be a function such that $\sigma(1) = 3$, $\sigma(2) = 2$, and $\sigma(3) = 1$.
- If you have n cards with labels 1 through n and you shuffle them, then you can let $\sigma(j)$ denote the label of the card in the jth position. Thus orderings of n cards are in one-to-one correspondence with permutations of n elements.
- One way to represent σ is to list the values $\sigma(1), \sigma(2), \ldots, \sigma(n)$ in order. The σ above is represented as $\{3, 2, 1\}$.

Permutation notation

- A **permutation** is a function from $\{1, 2, \ldots, n\}$ to $\{1, 2, \ldots, n\}$ whose range is the whole set $\{1, 2, \ldots, n\}$. If σ is a permutation then for each j between 1 and n, the the value $\sigma(j)$ is the number that j gets mapped to.
- For example, if $n = 3$, then σ could be a function such that $\sigma(1) = 3$, $\sigma(2) = 2$, and $\sigma(3) = 1$.
- If you have n cards with labels 1 through n and you shuffle them, then you can let $\sigma(j)$ denote the label of the card in the jth position. Thus orderings of n cards are in one-to-one correspondence with permutations of n elements.
- One way to represent σ is to list the values $\sigma(1), \sigma(2), \ldots, \sigma(n)$ in order. The σ above is represented as $\{3, 2, 1\}$.
- If σ and ρ are both permutations, write $\sigma \circ \rho$ for their composition. That is, $\sigma \circ \rho(j) = \sigma(\rho(j))$.

Cycle decomposition

- Another way to write a permutation is to describe its cycles:

Cycle decomposition

- Another way to write a permutation is to describe its cycles:
- For example, taking $n = 7$, we write $(2, 3, 5), (1, 7), (4, 6)$ for the permutation σ such that $\sigma(2) = 3, \sigma(3) = 5, \sigma(5) = 2$ and $\sigma(1) = 7, \sigma(7) = 1$, and $\sigma(4) = 6, \sigma(6) = 4$.

Cycle decomposition

- Another way to write a permutation is to describe its cycles:
- For example, taking $n = 7$, we write $(2, 3, 5), (1, 7), (4, 6)$ for the permutation σ such that $\sigma(2) = 3, \sigma(3) = 5, \sigma(5) = 2$ and $\sigma(1) = 7, \sigma(7) = 1$, and $\sigma(4) = 6, \sigma(6) = 4$.
- If you pick some j and repeatedly apply σ to it, it will "cycle through" the numbers in its cycle.

Cycle decomposition

- Another way to write a permutation is to describe its cycles:
- For example, taking $n = 7$, we write $(2, 3, 5), (1, 7), (4, 6)$ for the permutation σ such that $\sigma(2) = 3, \sigma(3) = 5, \sigma(5) = 2$ and $\sigma(1) = 7, \sigma(7) = 1$, and $\sigma(4) = 6, \sigma(6) = 4$.
- If you pick some j and repeatedly apply σ to it, it will "cycle through" the numbers in its cycle.
- Visualize this by writing down numbers 1 to n and drawing arrow from each k to $\sigma(k)$. Trace through a cycle by following arrows.

Cycle decomposition

- Another way to write a permutation is to describe its cycles:
- For example, taking $n = 7$, we write $(2, 3, 5), (1, 7), (4, 6)$ for the permutation σ such that $\sigma(2) = 3, \sigma(3) = 5, \sigma(5) = 2$ and $\sigma(1) = 7, \sigma(7) = 1$, and $\sigma(4) = 6, \sigma(6) = 4$.
- If you pick some j and repeatedly apply σ to it, it will "cycle through" the numbers in its cycle.
- Visualize this by writing down numbers 1 to n and drawing arrow from each k to $\sigma(k)$. Trace through a cycle by following arrows.
- Generally, a function f is called an **involution** if $f(f(x)) = x$ for all x.

Cycle decomposition

- Another way to write a permutation is to describe its cycles:
- For example, taking $n = 7$, we write $(2, 3, 5), (1, 7), (4, 6)$ for the permutation σ such that $\sigma(2) = 3, \sigma(3) = 5, \sigma(5) = 2$ and $\sigma(1) = 7, \sigma(7) = 1$, and $\sigma(4) = 6, \sigma(6) = 4$.
- If you pick some j and repeatedly apply σ to it, it will "cycle through" the numbers in its cycle.
- Visualize this by writing down numbers 1 to n and drawing arrow from each k to $\sigma(k)$. Trace through a cycle by following arrows.
- Generally, a function f is called an **involution** if $f(f(x)) = x$ for all x.
- A permutation is an involution if all cycles have length one or two.

Cycle decomposition

- Another way to write a permutation is to describe its cycles:
- For example, taking $n = 7$, we write $(2, 3, 5), (1, 7), (4, 6)$ for the permutation σ such that $\sigma(2) = 3, \sigma(3) = 5, \sigma(5) = 2$ and $\sigma(1) = 7, \sigma(7) = 1$, and $\sigma(4) = 6, \sigma(6) = 4$.
- If you pick some j and repeatedly apply σ to it, it will "cycle through" the numbers in its cycle.
- Visualize this by writing down numbers 1 to n and drawing arrow from each k to $\sigma(k)$. Trace through a cycle by following arrows.
- Generally, a function f is called an **involution** if $f(f(x)) = x$ for all x.
- A permutation is an involution if all cycles have length one or two.
- A permutation is "fixed point free" if there are no cycles of length one.

Outline

Remark, just for fun

Permutations

Counting tricks

Binomial coefficients

Problems

Outline

Fundamental counting trick

- n ways to assign hat for the first person. No matter what choice I make, there will remain $n-1$ ways to assign hat to the second person. No matter what choice I make there, there will remain $n-2$ ways to assign a hat to the third person, etc.

Fundamental counting trick

- n ways to assign hat for the first person. No matter what choice I make, there will remain $n-1$ ways to assign hat to the second person. No matter what choice I make there, there will remain $n-2$ ways to assign a hat to the third person, etc.
- This is a useful trick: break counting problem into a sequence of stages so that one always has the same number of choices to make at each stage. Then the total count becomes a product of number of choices available at each stage.

Fundamental counting trick

- n ways to assign hat for the first person. No matter what choice I make, there will remain $n-1$ ways to assign hat to the second person. No matter what choice I make there, there will remain $n-2$ ways to assign a hat to the third person, etc.
- This is a useful trick: break counting problem into a sequence of stages so that one always has the same number of choices to make at each stage. Then the total count becomes a product of number of choices available at each stage.
- Easy to make mistakes. For example, maybe in your problem, the number of choices at one stage actually *does* depend on choices made during earlier stages.

Another trick: overcount by a fixed factor

- If you have 5 indistinguishable black cards, 2 indistinguishable red cards, and three indistinguishable green cards, how many distinct shuffle patterns of the ten cards are there?

Another trick: overcount by a fixed factor

- If you have 5 indistinguishable black cards, 2 indistinguishable red cards, and three indistinguishable green cards, how many distinct shuffle patterns of the ten cards are there?
- Answer: if the cards were distinguishable, we'd have $10!$. But we're overcounting by a factor of $5!2!3!$, so the answer is $10!/(5!2!3!)$.

Outline

Outline

$\binom{n}{k}$ notation

- How many ways to choose an ordered sequence of k elements from a list of n elements, with repeats allowed?

$\binom{n}{k}$ notation

- How many ways to choose an ordered sequence of k elements from a list of n elements, with repeats allowed?
- Answer: n^k

$\binom{n}{k}$ notation

- How many ways to choose an ordered sequence of k elements from a list of n elements, with repeats allowed?
- Answer: n^k
- How many ways to choose an ordered sequence of k elements from a list of n elements, with repeats forbidden?

$\binom{n}{k}$ notation

- How many ways to choose an ordered sequence of k elements from a list of n elements, with repeats allowed?
- Answer: n^k
- How many ways to choose an ordered sequence of k elements from a list of n elements, with repeats forbidden?
- Answer: $n!/(n-k)!$

$\binom{n}{k}$ notation

- How many ways to choose an ordered sequence of k elements from a list of n elements, with repeats allowed?
- Answer: n^k
- How many ways to choose an ordered sequence of k elements from a list of n elements, with repeats forbidden?
- Answer: $n!/(n-k)!$
- How many way to choose (unordered) k elements from a list of n without repeats?

$\binom{n}{k}$ notation

- How many ways to choose an ordered sequence of k elements from a list of n elements, with repeats allowed?
- Answer: n^k
- How many ways to choose an ordered sequence of k elements from a list of n elements, with repeats forbidden?
- Answer: $n!/(n-k)!$
- How many way to choose (unordered) k elements from a list of n without repeats?
- Answer: $\binom{n}{k} := \frac{n!}{k!(n-k)!}$

$\binom{n}{k}$ notation

- How many ways to choose an ordered sequence of k elements from a list of n elements, with repeats allowed?
- Answer: n^k
- How many ways to choose an ordered sequence of k elements from a list of n elements, with repeats forbidden?
- Answer: $n!/(n-k)!$
- How many way to choose (unordered) k elements from a list of n without repeats?
- Answer: $\binom{n}{k} := \frac{n!}{k!(n-k)!}$
- What is the coefficient in front of x^k in the expansion of $(x+1)^n$?

$\binom{n}{k}$ notation

- How many ways to choose an ordered sequence of k elements from a list of n elements, with repeats allowed?
- Answer: n^k
- How many ways to choose an ordered sequence of k elements from a list of n elements, with repeats forbidden?
- Answer: $n!/(n-k)!$
- How many way to choose (unordered) k elements from a list of n without repeats?
- Answer: $\binom{n}{k} := \frac{n!}{k!(n-k)!}$
- What is the coefficient in front of x^k in the expansion of $(x+1)^n$?
- Answer: $\binom{n}{k}$.

Pascal's triangle

- Arnold principle.

Pascal's triangle

- Arnold principle.
- A simple recursion: $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$.

Pascal's triangle

- Arnold principle.
- A simple recursion: $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$.
- What is the coefficient in front of x^k in the expansion of $(x+1)^n$?

Pascal's triangle

- Arnold principle.
- A simple recursion: $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$.
- What is the coefficient in front of x^k in the expansion of $(x+1)^n$?
- Answer: $\binom{n}{k}$.

Pascal's triangle

- Arnold principle.
- A simple recursion: $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$.
- What is the coefficient in front of x^k in the expansion of $(x+1)^n$?
- Answer: $\binom{n}{k}$.
- $(x+1)^n = \binom{n}{0} \cdot 1 + \binom{n}{1}x^1 + \binom{n}{2}x^2 + \ldots + \binom{n}{n-1}x^{n-1} + \binom{n}{n}x^n$.

Pascal's triangle

- Arnold principle.
- A simple recursion: $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$.
- What is the coefficient in front of x^k in the expansion of $(x+1)^n$?
- Answer: $\binom{n}{k}$.
- $(x+1)^n = \binom{n}{0} \cdot 1 + \binom{n}{1}x^1 + \binom{n}{2}x^2 + \ldots + \binom{n}{n-1}x^{n-1} + \binom{n}{n}x^n$.
- Question: what is $\sum_{k=0}^{n} \binom{n}{k}$?

Pascal's triangle

- Arnold principle.
- A simple recursion: $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$.
- What is the coefficient in front of x^k in the expansion of $(x+1)^n$?
- Answer: $\binom{n}{k}$.
- $(x+1)^n = \binom{n}{0} \cdot 1 + \binom{n}{1}x^1 + \binom{n}{2}x^2 + \ldots + \binom{n}{n-1}x^{n-1} + \binom{n}{n}x^n$.
- Question: what is $\sum_{k=0}^{n} \binom{n}{k}$?
- Answer: $(1+1)^n = 2^n$.

Outline

Outline

More problems

- How many full house hands in poker?

More problems

- How many full house hands in poker?
- $13\binom{4}{3} \cdot 12\binom{4}{2}$

More problems

- How many full house hands in poker?
- $13\binom{4}{3} \cdot 12\binom{4}{2}$
- How many “2 pair” hands?

More problems

- How many full house hands in poker?
- $13\binom{4}{3} \cdot 12\binom{4}{2}$
- How many "2 pair" hands?
- $13\binom{4}{2} \cdot 12\binom{4}{2} \cdot 11\binom{4}{1}/2$

More problems

- How many full house hands in poker?
- $13\binom{4}{3} \cdot 12\binom{4}{2}$
- How many "2 pair" hands?
- $13\binom{4}{2} \cdot 12\binom{4}{2} \cdot 11\binom{4}{1}/2$
- How many royal flush hands?

More problems

- How many full house hands in poker?
- $13\binom{4}{3} \cdot 12\binom{4}{2}$
- How many "2 pair" hands?
- $13\binom{4}{2} \cdot 12\binom{4}{2} \cdot 11\binom{4}{1}/2$
- How many royal flush hands?
- 4

More problems

- How many hands that have four cards of the same suit, one card of another suit?

More problems

- How many hands that have four cards of the same suit, one card of another suit?
- $4\binom{13}{4} \cdot 3\binom{13}{1}$

More problems

- How many hands that have four cards of the same suit, one card of another suit?
- $4\binom{13}{4} \cdot 3\binom{13}{1}$
- How many 10 digit numbers with no consecutive digits that agree?

More problems

- How many hands that have four cards of the same suit, one card of another suit?
- $4\binom{13}{4} \cdot 3\binom{13}{1}$
- How many 10 digit numbers with no consecutive digits that agree?
- If initial digit can be zero, have $10 \cdot 9^9$ ten-digit sequences. If initial digit required to be non-zero, have 9^{10}.

More problems

- How many hands that have four cards of the same suit, one card of another suit?
- $4\binom{13}{4} \cdot 3\binom{13}{1}$
- How many 10 digit numbers with no consecutive digits that agree?
- If initial digit can be zero, have $10 \cdot 9^9$ ten-digit sequences. If initial digit required to be non-zero, have 9^{10}.
- How many ways to assign a birthday to each of 23 distinct people? What if no birthday can be repeated?

More problems

- How many hands that have four cards of the same suit, one card of another suit?
- $4\binom{13}{4} \cdot 3\binom{13}{1}$
- How many 10 digit numbers with no consecutive digits that agree?
- If initial digit can be zero, have $10 \cdot 9^9$ ten-digit sequences. If initial digit required to be non-zero, have 9^{10}.
- How many ways to assign a birthday to each of 23 distinct people? What if no birthday can be repeated?
- 366^{23} if repeats allowed. $366!/343!$ if repeats not allowed.