18.600: Lecture 12

Poisson random variables

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Outline

Poisson random variable definition

Poisson random variable properties

Poisson random variable problems

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Poisson random variable properties

Poisson random variable problems

Poisson random variables: motivating questions

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- **Key idea for all these examples:** Divide time into large number of small increments. Assume that during each increment, there is some small probability of thing happening (independently of other increments).

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- Can also change sign: $e^{-\lambda} = \lim_{n\to\infty}(1 - \lambda/n)^n$.

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- This is one way to *remember* the Poisson probability mass function. Just remember that it comes from Taylor expansion of e^{λ}.

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    - $e^{-\lambda}$ is approximate probability of all tails sequence.
    - λ^k comes from fact that *given* sequence with k heads is $(\lambda/n)^k$ times more probable than *given* sequence with zero heads.
    - $k!$ is "ordered vs. unordered overcount factor."

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- **Mnemonic:** binomial has variance npq, and Poisson is obtained by fixing $\lambda = np$ and taking $q \to 1$, so Poisson has variance $\lambda = np$. It's like npq without the q.

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- Then $\mathrm{Var}[X] = E[X^2] - E[X]^2 = \lambda(\lambda+1) - \lambda^2 = \lambda$.

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- A casino deals one million five-card poker hands per year. Approximate the probability that there are exactly 2 royal flush hands during a given year.
- Expected number of royal flushes is $\lambda = 10^6 \cdot 4/\binom{52}{5} \approx 1.54$. Answer is $e^{-\lambda}\lambda^k/k!$ with $k = 2$.

18.600 Probability and Random Variables
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