

### 18.700 Problem Set 1

Due in class Tuesday September 17; late work will not be accepted. Your work on graded problem sets should be written entirely on your own, although you may consult others before writing. I have written a few words about what kind of solution I am looking for; of course you can get more hints about that (for future use) from the posted solutions.

1. (3 points) Consider the set of complex numbers

$$G = \{a + bi \mid a, b \in \mathbb{Q}\}.$$

(The  $G$  stands for *Gauss*; these numbers might be called *Gaussian rational numbers*, although I don't know if they actually are.) Is  $G$  a field (with the same addition and multiplication operations as in  $\mathbb{C}$ )? For a question like this, you should either explain why all the axioms for a field are satisfied (you can assume that they hold for  $\mathbb{C}$ ), or else explain why one of the axioms fails. A few sentences could be enough to write.

2. (3 points) Consider the set of complex numbers

$$M = \{re^{2\pi i\theta} \mid r, \theta \in \mathbb{Q}\}.$$

Is  $M$  a field?

3. (3 points) The vector space  $V = (\mathbb{F}_2)^2$  has exactly four vectors  $(0, 0)$ ,  $(0, 1)$ ,  $(1, 0)$ , and  $(1, 1)$ ; so  $V$  has exactly  $2^4 = 16$  subsets. How many of these 16 subsets are linearly independent? How many bases does  $V$  have? For a question like this, you might write some words explaining why some kinds of subset cannot possibly be linearly independent (“the vector  $(1, 1)$  is in the pay of Big Oil, and so cannot be part of any linearly independent set”). After this you might be left with just a few cases; you could perhaps say a few words about why each of these is or is not linearly independent.

4. (3 points) Axler, page 19, exercise 5.
5. (3 points) Axler, page 19, exercise 15.
6. (3 points) Axler, page 35, exercise 1.
7. (3 points) Axler, page 35, exercise 3.

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