18.700 Problem Set 2

Due in class Monday September 24; late work will not be accepted. Your work on graded problem sets should written entirely on your own, although you may consult others before writing.

1. (3 points) Let V be the vector space of polynomials of degree at most five with real coefficients. Define a linear map

$$T: V \to \mathbb{R}^3, \quad T(p) = (p(1), p(2), p(3)).$$

That is, the coordinates of the vector $T(p)$ are the values of p at 1, 2, and 3.
a) Find a basis of the null space of T.
b) Find a basis of the range of T.

2. (3 points) Let V be the vector space of polynomials of degree at most 999 with real coefficients. Define a linear map

$$T: V \to \mathbb{R}^{100}, \quad T(p) = (p(1), p(2), \ldots, p(100)).$$

a) Find the dimension of the null space of T.
b) Find the dimension of the range of T.

3. (6 points) Let V be the vector space of polynomials of degree at most 99 with real coefficients. Define a linear map

$$T: V \to \mathbb{R}^{1000}, \quad T(p) = (p(1), p(2), \ldots, p(1000)).$$

a) Find the dimension of the null space of T.
b) Find the dimension of the range of T.
c) (This one is hard.) Is the vector $(0, 1, 0, 1, 0, 1, \ldots, 0, 1)$ in the range of T? That is, is there a polynomial of degree at most 99 whose values at $1, 2, \ldots, 1000$ alternate between 0 and 1?

4. (2 points) Axler, page 36, exercise 12.

5. (2 points) Axler, page 36, exercise 16.

6. (2 points) Axler, page 36, exercise 17.

7. (2 points) Axler, page 59, exercise 7.

18.700 Linear Algebra
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