Hint on Spherical Coordinates

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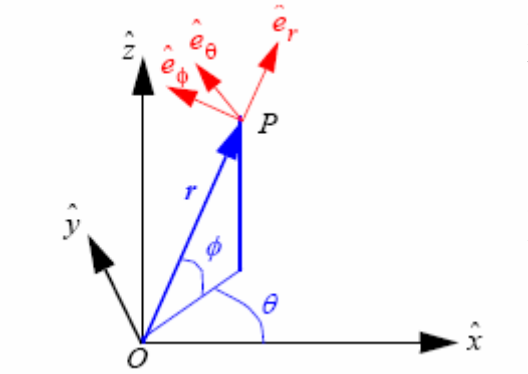
There is a reference to spherical coordinates Problem 2 in Problem Set #1. Here is a clarification on how to proceed.

The problem gives you the velocity of the aircraft in the *x* direction. The dimensions are given to you in Cartesian coordinates, which are natural for the pilot. However, the plane is being tracked by a radar. The natural coordinates for a radar are spherical. The problem asks you to express the velocity of the aircraft in terms of unit vectors attached to the spherical coordinate system.



Here's something that might help you. The diagram below shows $\hat{e}_R$, $\hat{e}_\theta$ and $\hat{e}_\phi$, the unit vectors in the spherical system, with respect to those in Cartesian coordinates, namely $\hat{i}$, $\hat{j}$ and $\hat{k}$. The two can be related to each other with some basic trigonometry and projections. For example:

$$\hat{j} = \hat{e}_R \cos\theta\cos\phi - \hat{e}_\phi \cos\theta\sin\phi + \hat{e}_\theta \sin\theta.$$

You can come up with similar expressions for $\hat{i}$ and $\hat{k}$ in terms of $\hat{e}_R$, $\hat{e}_\theta$ and $\hat{e}_\phi$. Once you do, you have the necessary machinery to answer Question 2.