

Sloan School of Management Massachusetts Institute of Technology 15.010/15.011 – Fall 2004 Berndt, Chapman, Doyle and Stoker

SOLUTIONS TO HOMEWORK SET #2

1.

a. In this problem, we have two different supply functions (domestic producers and foreign producers) and one demand function. To find the equilibrium we need to combine both supply functions into one "effective" supply and find its intersection with the demand.

Let's construct the "effective" supply function: Think of the supply function as the minimum price that a producer is asking to deliver a given quantity of sugar.

From Figure 1, we can observe that are some US producers that are willing to deliver sugar for less than 10 cents/lb. (the world price).

There are also US producers that are asking for a price higher than 10 cents/lb. However, given that US consumers can purchase sugar from foreign producers at 10 cents/lb, they will not be interested in this relatively expensive US domestic supply.

Therefore, our "effective" supply will be formed by those US producers willing to sell sugar for less than 10 cents/lb and for foreign producers selling at 10 cents/lb. The "effective" supply is the bold line in Figure 1.



At this point, it should be clear that the equilibrium price will be equal to the world price of US\$ 10 cents/lb. At this price, the domestic demand will be: $Q = 60 - \frac{2}{3} \cdot 10 = 53.3$ billion lb.

The quantity supplied domestically will be: Q = P = 10 billion lb. Imports under free trade will be: 53.3-10 = 43.3 billion lb.

The consumer surplus will be $\frac{1}{2} \cdot (90-10) \cdot 53.3 = 2,133$.

Be aware of the units: you are multiplying $(US$ cents/lb) (billion lb) = billion US$ cents! <math>\Rightarrow$ You have to divide the result by 100 to have US\$ billion

Thus, the consumer surplus will be US\$ 21.33 billion. The producer surplus will be $\frac{1}{2} \cdot 10 \cdot 10 = 50$. Then, the producer surplus will be US\$ 0.5 billion

b. In this scenario, the world price is not relevant (there are no imports), so the US demand should be equal to the US domestic supply:

 $Q^{d} = Q^{s}$ $60 - {}^{2}/_{3} \cdot P = P$ $P = 60 \cdot 3/5 = 36 \text{ cents/lb}$ Q = 36 billion lb.

Without imports, the quantity demanded will be 36 billion lb. and the equilibrium price will be 36 cents/lb



The consumer surplus is $\frac{1}{2} \cdot (90-36) \cdot 36 = 972$ billion cents \Rightarrow US\$ 9.72 billion The producer surplus is $\frac{1}{2} \cdot 36 \cdot 36 = 648$ billion cents \Rightarrow US\$ 6.48 billion The deadweight loss could be calculated in two ways:

- as the area of the triangle formed by the US domestic supply, the US demand and the world price: ½ ·(36-10)(53.3-10) = 563.3 billion cents ⇒ US\$ 5.63 billion
- as the difference between the surplus in both scenarios: (21.33+0.5)-(9.72+6.48) = US\$ 5.63 billion
- c. In this question, we can build on the notion of "effective supply we used for the answer to part a. The subsidy benefits domestic production in 15 cents/lb. You can think of that as a reduction in costs or a reduction in the minimum price that they will ask for any given quantity. Then, the subsidy will be reflected in a downward movement of the US domestic supply, as presented in Figure 3.

The subsidy enables more domestic producers to offer sugar for less than 10 cents/lb. Given that US consumers can purchase sugar from foreign producers at 10 cents/lb, they will not be interested in those US producers who, even after the subsidy, ask for more than the world price.

Therefore, our "effective" supply will be formed by those US producers willing to sell sugar for less than 10 cents/lb and for foreign producers selling at 10 cents/lb. The "effective" supply is the bold line in Figure 3.



The equilibrium price will return to the world price of 10 cents/lb. At this price, the quantity demanded will be 53.3 billion lb. The domestic supply is now benefiting from the subsidy, so Q = P+15 = 10+15 = 25 billion lb. Imports will be 53.3 - 25 = 28.3 billion lb.

The consumer surplus will be $\frac{1}{2} \cdot (90\text{-}10) \cdot 53.3 = 2,133$ billion cents \Rightarrow US\$ 21.33 billion. (*Note that this is the same result as part a.*) The producer surplus (A+B+C) will be $\frac{1}{2} \cdot (10\text{+}15) \cdot 25 = 312.5$ billion cents \Rightarrow US\$ 3.125 billion The amount of the subsidy is $15 \cdot 25 = 375$ billion cents \Rightarrow US\$ 3.75 billion

Figure 3 shows the amount paid by the government as a subsidy (B+C+D):

- B and C are transfers from the government to producers and increase producer surplus compared to the result in part a.
- The government also pays D as part of the subsidy. However, this transfer is not part of producer surplus because it only offsets the additional costs of domestic producer compared with foreign producers.

The deadweight loss in this situation corresponds to the amount paid by the government that is not transferred to producers, but covers additional costs of producers that are less efficient than world producers. The deadweight loss is equal to the area of $D = \frac{1}{2} \cdot (25-10) \cdot (25-10) = 112.5$ billion cents = US\$ 1.125 billion.

We arrive at the same result if we compare the total surplus of the part a. (21.33+0.5) and the total surplus of part c.(21.33+3.125-3.75). The difference is US\$ 1.125 billion.

(in US\$ million)	Free Trade	No Trade	Subsidy
Consumer Surplus	21,333	9,720	21,333
Producer Surplus	500	6,480	3,125
Government			-3,750
Total Welfare	21,833	16,200	20,708

d. We can compare the situation under the three previous scenarios:

Clearly, consumers are much better with free trade than without it. Therefore, consumer organizations, such as soft drink manufacturers and candy makers will lobby in favor of free trade in sugar. Producers benefit from trade barriers or subsidies and, in consequence, will lobby in order to obtain such privileges.

e. Let Q equal the total quantity supplied of sugar and q equal the quota that the government should set in order to have a domestic production of 25 billion lb. The total quantity traded in the market (Q) will come from imports (q) or from domestic production (25 billion lb), so we can formulate the following relationship for equilibrium:

$$Q = q + 25 ... (I)$$

Also, in Figure 4 we can observe that the market will have a kinked supply curve where:

Q = P if P < 10 cents/lb = P + q if P > 10 cents/lb

Solving for P and substituting for Q using (I): P = Q - q = 25 cents/lb

Using this price in the demand equation, the total quantity demanded in equilibrium will be: $Q = 60 - \frac{2}{3} \cdot 25 = 43.3$ billion lb. The quota will be 43.3 - 25 = 18.3 billion lb.



We will use Figure 4 to describe the consumer and producer surplus. The consumer surplus is the area of the triangle $E = \frac{1}{2} \cdot (90-25) \cdot 43.33 = 1,408.3$ billion cents \Rightarrow \$14.08 billion

The domestic producer surplus will be A+B+D:

- A: $\frac{1}{2} \cdot 10 \cdot 10 = 50$ billion cents = US\$ 0.5 billion
- B: (25-10)·10 = 150 billion cents = US\$ 1.5 billion
- D: $\frac{1}{2} \cdot (43.3 28.3) \cdot (25 10) = \frac{1}{2} \cdot (15)(15) = 112.5$ billion cents \Rightarrow \$ 1.125 billion

Then, the domestic producer surplus is US\$ 3.125 billion

The transfer to international producers (Area C in Figure 4) is $18.3 \cdot (25-10) =$ 275 billion cents \Rightarrow US\$ 2.75 billion. This transfer does not occur in the scenario of a subsidy (part c.)

Finally, we can compare the effects of a quota system with the effects of a subsidy in the following table:

	Subsidy	Import Quota
Domestic production (billion lb)	25	25
Consumer Surplus (US\$ million)	21,333	14,083
Producer Surplus (US\$ million)	3,125	3,125
Government	-3,750	
Total Welfare	20,708	17,208

The scenario of the import quota was constructed in order to have the same domestic production (25 billion lb.) With the subsidy, the effective price received was \$.25. Here, the quota pushes the market price up to \$.25, so the producer surplus is the same. Consumer surplus, however, is much lower. The total surplus for the society is also lower in the quota scenario than in the subsidy scenario.

f. By this time, you are familiar with the idea of an "effective" supply that combines the domestic production of sugar, the domestic supply of HFCS and the imports quota of 15bn pounds. Considering both the quota and the HFCS supply, we have a kinked supply curve, with the following pattern:

Р	=	Q	if Q < 10
	=	10	if $10 \le Q \le 25^{(1)}$
	=	Q-15 ⁽²⁾	if $25 \le Q \le 30^{(3)}$
	=	15	if $30 \le Q < 40^{(4)}$
	=	Q-25 ⁽⁵⁾	if $Q \ge 40$

⁽¹⁾ [Domestic supply with cost lower than world price]+[imports]= 10+15 = 25⁽²⁾ Domestic producers supply in this range:

[total supply] - [imports] = Q - 15

⁽³⁾ Domestic producers will supply sugar until its cost is equal to the marginal cost of producing HFCS. The upper limit of this range occurs when:

 $Q-15 = 15 \Rightarrow Q = 30$ ⁽⁴⁾ Take the 30 from ⁽³⁾ and add the HFCS capacity (10 billion lb.) ⁽⁵⁾ As in ⁽²⁾, dometic producers supply sugar in this range:

[total supply] - [imports] - [HFCS] = O - 15 - 10 = O - 25

The "effective" supply is the bold line in Figure 5.

The equilibrium will occur when supply equals demand: P=90 - 1.5Q, then:

Q-25 = 90 - 1.5Q

Q = 46 billion lb.

The quantity demanded is 46 billion lb, composed of 15 billion lb. of imported sugar, 10 billion lb. of HFCS and 11 billion lb. of domestic sugar production The market price is 46 - 25 = 21 cents/lb.

The surplus analysis is based on Figure 5.

The consumer surplus is equal to $A = \frac{1}{2} \cdot (90-21) \cdot 46 = 1,587$ billion cents = US\$ 15.87 billion

The domestic sugar producer surplus is equal to B+C+E+F+H

- Area B: $\frac{1}{2} \cdot 10 \cdot 10 = 50$ billion cents
- Area C: (21-10) 10 = 110 billion cents
- Area E: $\frac{1}{2} \cdot (15-10) \cdot (30-25) = 12.5$ billion cents
- Area F: (21-15) (30-25) = 30 billion cents
- Area H: $\frac{1}{2} \cdot (21-15) \cdot (46-40) = 18$ billion cents

Then, the domestic sugar producer surplus is 220.5 billion cents or US\$ 2.205 billion



HFCS producer surplus is $G = (21-15) \cdot (40-30) = 60$ billion cents or US\$ 0.6 billion

World sugar producers surplus is $D = (21-10) \cdot (25-10) = 165$ billion cents or US\$ 1.65 billion

The deadweight loss is I+J+K+L+M

- Area I: $\frac{1}{2}$ (21-15)•(46-40) = 18 billion cents
- Area J: $\frac{1}{2} \cdot (15-10) \cdot (30-25) = 12.5$ billion cents
- Area K: (15-10)·(40-30) = 50 billion cents
- Area L: (15-10)·(46-40) = 30 billion cents
- Area M: $\frac{1}{2}$ (21-10) (53.3-46) = 40.3 billion cents

Then, the deadweight loss is 150.83 billion cents or US\$ 1.5083 billion

2. In order to understand the different statements, it is useful to summarize the information in the following table:

Units	Revenue	Variable	Fixed	Total	Profit
	(P.Q)	Cost	Cost	Cost	
from table	given	from table	given	VC+FC	TR - TC
100	16,000	5,000	30,000	35,000	(19,000)
200	32,000	15,000	30,000	45,000	(13,000)
300	48,000	24,000	30,000	54,000	(6,000)
400	64,000	33,000	30,000	63,000	1,000
500	80,000	43,000	30,000	73,000	7,000
600	92,000	54,000	30,000	84,000	8,000
700	104,000	67,000	30,000	97,000	7,000
800	116,000	83,000	30,000	113,000	3,000
900	128,000	103,000	30,000	133,000	(5,000)
1,000	140,000	128,000	30,000	158,000	(18,000)

After examining the table, we can conclude that the maximum profit is obtained producing 600 units. If the firm produces less, it is losing the opportunity of additional gains from selling up to 600 units. If the firm produces more than 600 units, the increase in costs due to the additional units will be higher than the revenue extracted from the additional units sold.

Let us look at two examples:

- i) **Production of 500 units:** it is not the optimal situation, because increasing from 500 to 600 means additional revenue of \$12,000 with additional costs of only \$11,000.
- **ii) Production of 700 units:** it is not the optimal situation. Moving the production from 600 to 700 increases revenues by \$12,000 and costs by \$13,000. The result is a loss of \$1,000 in the last 100 units.

This idea of additional costs incurred when the number of units produced is increased is known as Marginal Cost. Its general formula is:

Marginal costs: $MC = \Delta Total Costs(Q) / \Delta Q$

You can compare the Marginal Cost and the Average Cost (Total Cost per unit) in the following table:

Units	Total	Average	Marginal	Price
	Cost	Cost	Cost	
	VC+FC	\$/Unit	\$/Unit	given
100	35,000	350	350	160
200	45,000	225	100	160
300	54,000	180	90	160
400	63,000	158	90	160
500	73,000	146	100	160
600	84,000	140	110	120
700	97,000	139	130	120
800	113,000	141	160	120
900	133,000	148	200	120
1,000	158,000	158	250	120

As a conclusion, setting output up to the point where price is greater than or equal to marginal cost (in this case, 600 units) results in the maximum profit level (\$8,000).

To respond to the arguments presented:

- The company should not turn the order down because profits are positive between 400 and 800 units.
- Although the statement that Unity Car would have a profit producing 500 units is right, the argument that Unity Car should stop at 500 does not maximize profits. The opinion is based on the comparison of Price and Average Costs. The optimal level is reached by comparing the Price and the Marginal Cost of producing additional units, not the Average costs, and the above chart shows that P>MC up to and including 600 units.
- The last argument again compares Average Costs to prices, rather than focusing on the relationship between Marginal Costs and prices. By doing so, the pricing structure is different from the optimal one. If, however, Packard did change the pricing structure, producing at 1000 would not be optimal as the MC at 1000 units (= \$250) exceeds the \$160 price. Instead, the new optimal production amount would be 800 where MC=\$160.

a. To answer this question we will use some of the notions discussed in the answer to question 1.

First, we have three producers and one demand function. To calculate the equilibrium price and quantity we will need to combine the three plants in an "effective" supply curve and find the intersection with the demand function. Second, it will be useful again to think of the supply function as the minimum price that a producer is asking to deliver a given quantity.

Taking into account the previous two statements we can proceed to construct the "effective" supply of electricity. The plant that will ask for the lowest price is the one with the lowest cost, i.e. the coal plant. After supplying 4.6 GW-h, this plant reaches its maximum capacity. To supply more electricity, it is required to choose between oil and gas. Once again, the lowest cost producer (i.e. the oil plant) enters before higher cost producers, until it reaches its maximum capacity. Only if the electricity required exceeds 6.6 GW-h, the gas plant will produce.

You can see the entire schedule in Figure 6



The special form of this supply curve forces us to adopt a different approach to solve the problem. We know that the price will take one of three values:

$$P = 1310 if Q \le 4.6 = 1730 if 4.6 < Q \le 6.6 (= 4.6+2.0) = 1860 if 6.6 < Q \le 9.3 (= 6.6+2.7)$$

3.

However, we do not know at which level the supply will intersect the demand. One way to find the equilibrium for each season is replacing each of the three values of P in the demand equation and check if the resulting Q belongs to the intervals presented above.

<u>Dry Season:</u> $Q = 14.25 - 0.005 \cdot P$

- If P = 1310, then Q should be less than 4.6. But, if we replace P in the demand equation \Rightarrow Q is 14.25- 0.005 · 1310 = 7.7 > 4.6 (X)
- If P = 1730, then Q should take a value between 4.6 and 6.6. Replacing P in the demand equation ⇒ Q is 14.25-0.005 1730 = 5.6, which is in the range of Oil production (✓)

Then, the price for the dry season is C1,730/GW-h and the peak demand is 5.6 GW-h.

The consumer surplus will be $\frac{1}{2} \cdot (2850-1730) \cdot 5.6 = C$ \$ 3,136 during the peak hour The coal plant surplus will be $(1730-1310) \cdot 4.6 = C$ \$ 1,932 during the peak hour The oil plant has no surplus (Marginal cost = Price). The gas plant does not produce

<u>Wet Season:</u> $Q = 18.3 - 0.005 \cdot P$

- If P = 1310, then Q should be less than 4.6. But, if we replace P in the demand equation \Rightarrow Q is $18.3 0.005 \cdot 1310 = 11.75 > 4.6$ (X)
- If P = 1730, then Q should take a value between 4.6 and 6.6. Replacing P in the demand equation \Rightarrow Q is 18.3- 0.005 · 1730 = 9.65 > 6.6 (X)
- If P = 1860, then Q should take a value between 6.6 and $9.3 \Rightarrow Q$ is 18.3- 0.005 \cdot 1860 = 9 < 9.3 (\checkmark)

Then, the price for the wet season is C\$1,860/GW-h and the peak demand is 9 GW-h. The consumer surplus will be $\frac{1}{2} \cdot (3660-1860) \cdot 9 = C$ \$ 8,100 during the peak hour The surplus of the coal plant will be (1860-1310) $\cdot 4.6 = C$ \$ 2,530 during the peak hour

The surplus of the oil plant will be $(1860-1730)\cdot 2 = C$ 260 during the peak hour. The gas plant has no surplus (Marginal cost = Price). Figure 7 describes the demand and supply for electricity:



b. The marginal cost of the oil plant is now $1730 \cdot 1.2 = C$ 2076/GW-h.

This changes the order in which the plants will enter production. Now, the gas plant is second and the oil plant is third. The effect on the market is presented in Figure 8.



Consumers will have to pay a higher price for electricity during both the Dry season (1860 vs. 1730, a 7.5% increment) and the Wet season (2076 vs. 1860, a 11.6% increment).

Coal plant increases its surplus because of the higher price paid by consumers. Gas plant has surplus during the Wet season (it was zero in the previous scenario), so it improves its results.

Oil plant loses because now it does not have any surplus (it does not produce during the dry season and only covers its costs during the wet season).

The previous evidence supports the idea that both the coal and gas plant will be in favor of the tax. On the other side, the oil plant and the consumers will oppose this measure.

4.

- a. The user cost of capital (UCC) of the "Daily Bugle" should change after the mayor's decision, because the UCC reflects the best alternative use of the asset. The new regulation allows more uses of the Blue House, and the highest-valued alternative use will now be chosen from a larger set of options. The only case in which the user cost of capital will remain the same is if, after examining all the new alternatives, they decide that the best possible alternative use for the building did not change.
- b. The activists' argument is wrong because they ignore opportunity cost. The fact that the Bugle pays no rent does not mean its costs are low. The opportunity cost of staying in the Blue House is the highest valued alternative use for the property. For example, say that the highest valued alternative use came from leasing the Blue House for other uses. These foregone lease payments should be incorporated into the cost of keeping the Bugle in the Blue House.
- 5. TRUE

When a firm experiences economies of scale, its average cost is decreasing with output. Economies of scale consider the average cost of production for the entire amount of output produced, whereas marginal cost considers only the last unit produced. If average cost is decreasing, then marginal cost must be below average costs, although marginal cost can still be increasing (see the graph on page 226 of the textbook).