15.401 Recitation

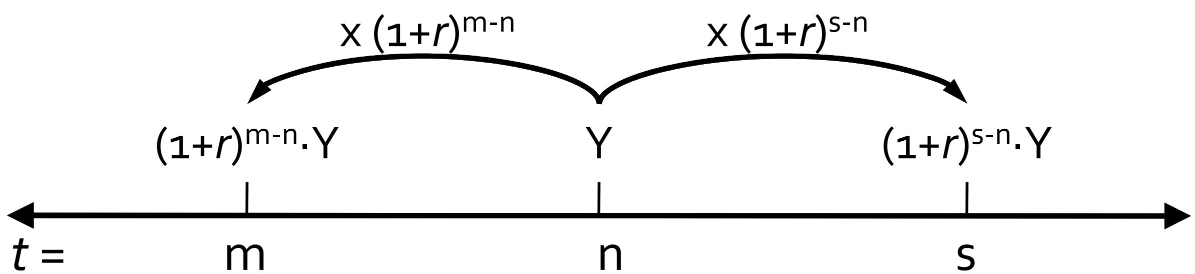
1: Present Value

Learning Objectives

- Review of Concepts
  - Compounding/discounting
  - PV/FV
  - Real vs. nominal rate
  - Annuities and perpetuities
- Examples
  - CD
  - Auto loan
  - Scholarship fund
  - Project planning

Review: Compounding / Discounting

- We can…
  - move money forward in time by **compounding**.
  - move money backward in time by **discounting**.



- Note:
  - Only relative time matters
  - Multiplying by $(1+r)^{m-n}$ = dividing by $(1+r)^{n-m}$.

Review: APR vs. EAR

- **Annual percentage rate** (**APR**) vs. **equivalent annual return (EAR):**

$$\text{EAR} = \left(1 + \frac{\text{APR}}{N}\right)^N - 1$$ (N = comp. freq.)

- Note:
  - **always** use the EAR when compounding and discounting
  - Due to interest compounding, the EAR is higher than the APR whenever the compounding frequency is higher than once a year.

Continuous Compounding (optional)

- Given a fixed APR, higher compounding frequency leads to higher EAR. Suppose we take compounding frequency to infinity, then

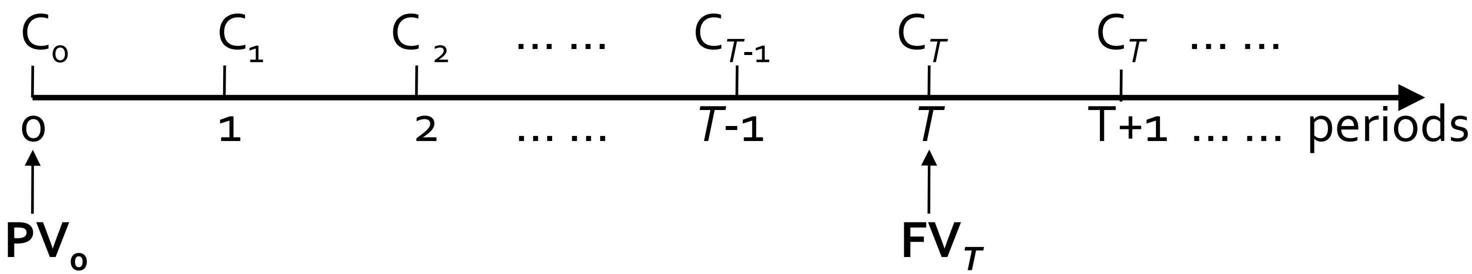
$$\text{EAR}_{\infty} = \lim_{n \to \infty} \left(1 + \frac{\text{APR}}{N}\right)^{N} - 1 = e^{\text{APR}} - 1.$$

$$\left(e = 2.71828183\ldots\right)$$

- The continuously compounded EAR is the highest possible EAR for a given APR.

Review: PV / FV

- Cash flow:



- Present value (PV):

$$\mathrm{PV}_0 = \mathrm{C}_0 + \frac{\mathrm{C}_1}{(1+r)^1} + \frac{\mathrm{C}_2}{(1+r)^2} + \cdots$$

- Future value (FV) :

$$\mathrm{FV}_T = \mathrm{C}_0(1+r)^T + \mathrm{C}_1(1+r)^{T-1} + \cdots$$
$$+ \mathrm{C}_T(1+r)^0 + \mathrm{C}_{T+1}(1+r)^{-1} + \cdots$$

Review: Nominal vs. Real Interest Rate

- Nominal-real interest rate conversion:

$$1 + r_{\text{real}} = \frac{1 + r_{\text{nominal}}}{1 + i}$$

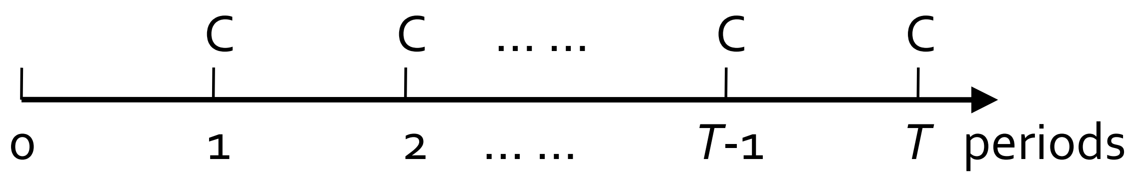
- Nominal-real cash flow conversion:

$$C_{\text{real}} = \frac{C_{\text{nominal}}}{1 + i}$$

- When you discount or compound,
  - Either use the **nominal** cash flow and the **nominal** interest rate
  - Or use the **real** cash flow and the **real** interest rate
  - **Do not** mix and match

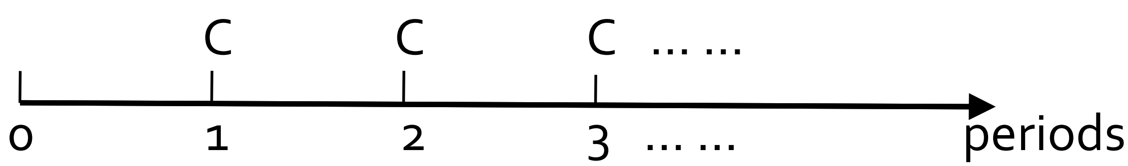
Review: Annuity/Perpetuity

- Annuity:



$$PV_0 = \frac{C}{r}\left[1 - \frac{1}{(1+r)^T}\right]$$

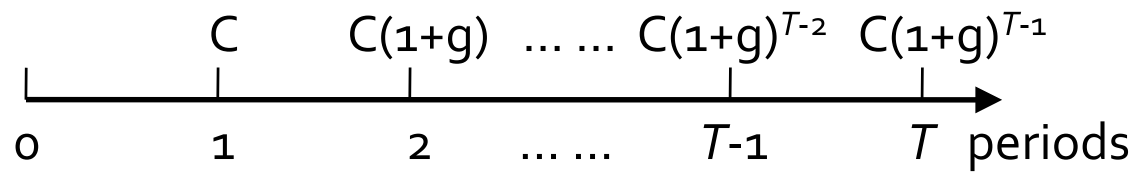
- Perpetuity:



$$PV_0 = \frac{C}{r}$$

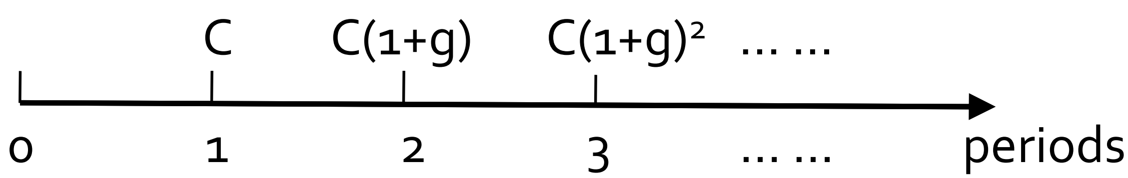
Review: Growing Annuity/Perpetuity

- Growing Annuity:



$$PV_0 = \frac{C}{r-g}\left[1 - \frac{(1+g)^T}{(1+r)^T}\right].$$

- Growing Perpetuity ($r > g$):



$$PV_0 = \frac{C}{r-g}.$$

Example 1: CD

- You can invest $10,000 in a CD offered by your bank. The CD matures in 5 years and the bank quotes you a rate of 4.5%. How much will you have in 5 years, if the 4.5% is
  a) EAR
  b) Quarterly APR
  c) Monthly APR

Example 1: CD

- Answer:
  a) $10{,}000 \times (1.045)^5 = \$12{,}461.82$

  b) $$r_{\text{EAR}} = \left(1 + \frac{0.045}{4}\right)^4 = 1.04576$$

  $$10{,}000 \times (1.04576)^5 = \$12{,}507.51$$

  c) $$r_{\text{EAR}} = \left(1 + \frac{0.045}{12}\right)^{12} = 1.04594$$

  $$10{,}000 \times (1.04594)^5 = \$12{,}517.96$$

Example 2: Auto Loan

- You would like to buy a new car for $22,000. The dealer requires a down payment of $10,000 and offers you 6% APR financing (compounded monthly) for 5 years for the remaining balance. What is your monthly payment?

Example 2: Auto Loan

- Answer: let C be the monthly payment, then

$$22000 = \frac{C}{0.06/12}\left[1 - \frac{1}{(1+0.06/12)^{12\times 5}}\right] + 10000.$$

$$C = \$231.99.$$

Example 3: Scholarship Fund

- You would like to establish a scholarship fund that will help outstanding students with financial difficulties pay their college tuition.
  - Starting today, you hope to give 50 students $20,000 each in today's money (i.e., adjusted for inflation) every year.
  - The effective nominal interest rate is 5%/yr.
  - Inflation is 2%/yr.
- How much money do you need now if you want the fund to last forever?

Example 3: Scholarship Fund

- Answer:
  - Method 1: nominal amount + nominal interest rate

$$1\text{m} + \frac{1\text{m} \times 1.02}{1.05 - 1.02} = 35\text{m}.$$

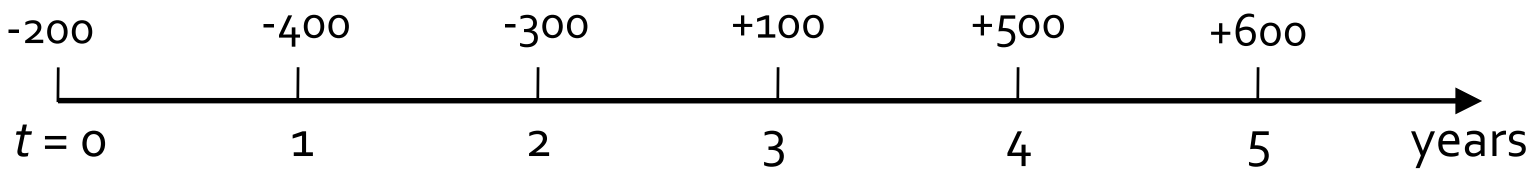
  - Method 2: real amount + real interest rate

$$r_{\text{real}} = \frac{1.05}{1.02} - 1 = 2.9412\% \quad \Rightarrow \quad 1\text{m} + \frac{1\text{m}}{0.029412} = 35\text{m}.$$

  - Note: same answer!
- You need $35 million today.

Example 4: Project Planning

- GeneriCorp is considering whether or not to expand into a new market. The company faces the following cash flow (in $million) if it decides to expand:



- A committee appointed by the CEO determined that the appropriate discount rate is 9%. Should the company take on the expansion project?

Example 4: Project Planning

- Answer:

$$\text{NPV} = -200 - \frac{400}{1.09} - \frac{300}{1.09^2} + \frac{100}{1.09^3} + \frac{500}{1.09^4} + \frac{600}{1.09^5}$$
$$= \$1.91\text{m}.$$

- Positive NPV = take the project; though NPV is dangerously close to zero.

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