15.401 Recitation

5: Options

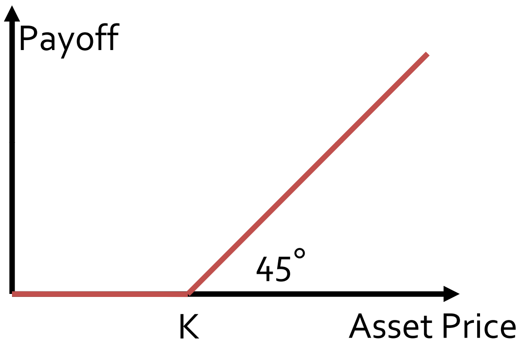
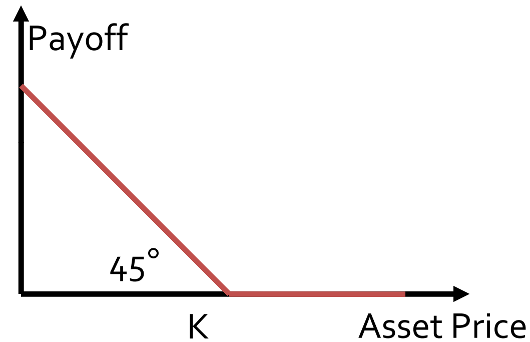
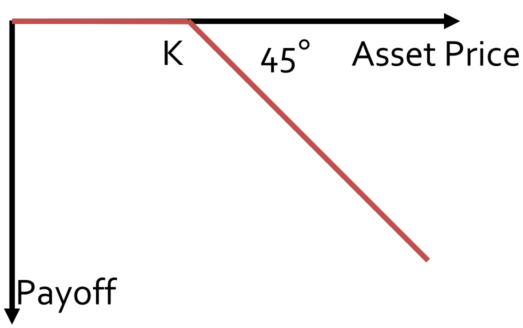
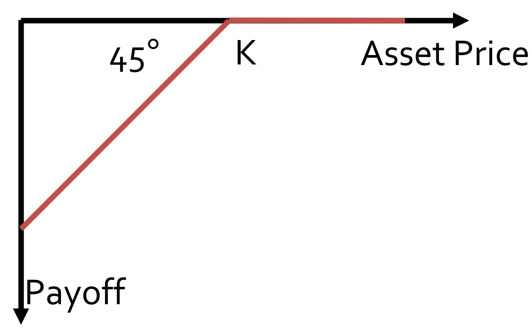
Learning Objectives

- Review of Concepts
  - Payoff profile
  - Put-call parity
  - Valuation of options
  - Binomial tree
- Examples
  - Payoff replication
  - Arboreal Corporation

Review: elements of a call/put option

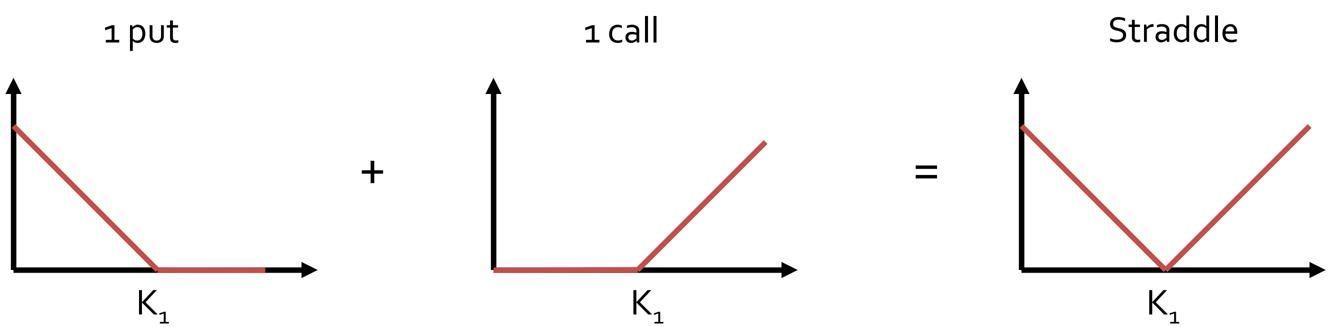
- Type:
  - Call: holder has the right but not the obligation to buy
  - Put: holder has the right but not the obligation to sell
- Quantity of the underlying asset:
  - Usually one share of stock with current price *S*
- Strike/exercise price (*K*)
- Expiration date (*T*)
- Style:
  - European: can only be exercised at *T*
  - American: can be exercised at any time between 0 and *T*.

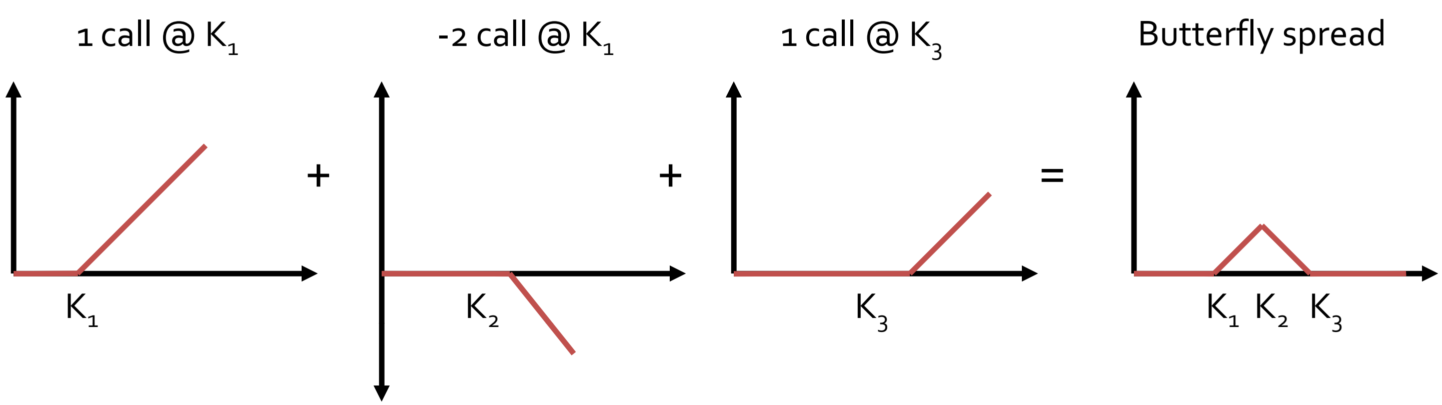
Review: payoff profile

	Call	Put
Long		
Short		

Review: payoff profile

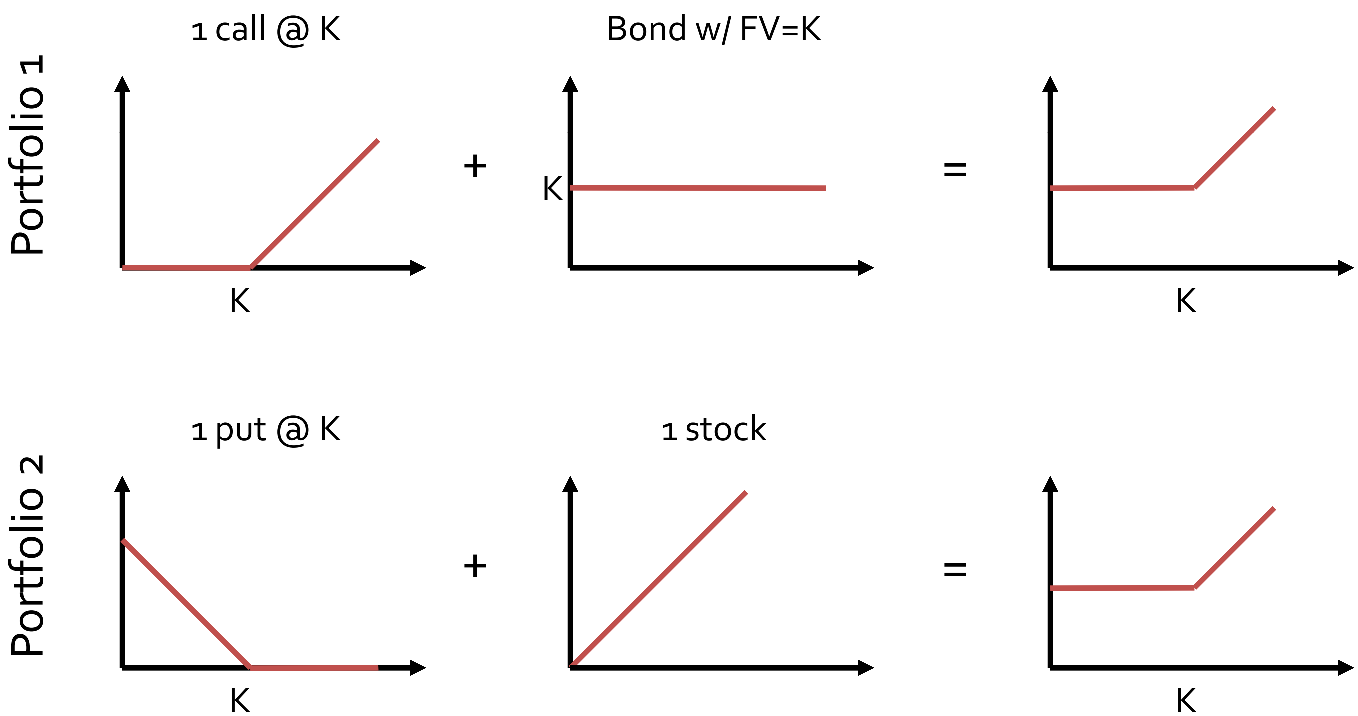
- The payoff of a portfolio of options is the sum of payoffs of the individual components:





Review: put-call parity

- Two portfolios with identical payoffs



Review: put-call parity

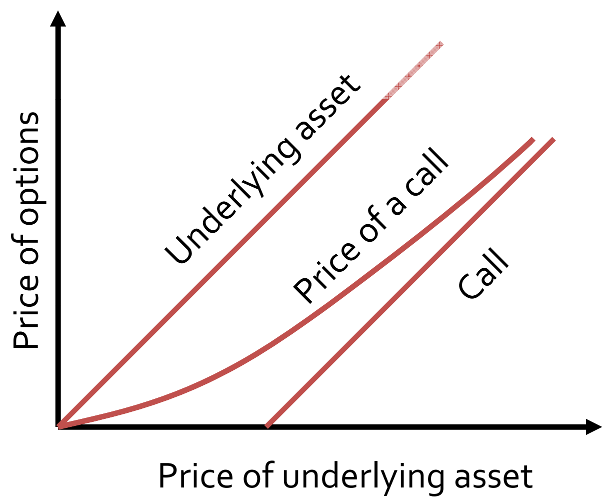
- No arbitrage implies that the two portfolios must have the same cost:

$$C + PV(K) = P + S$$

$$C + \frac{K}{(1+r)^T} = P + S$$

- This is the **put-call parity**.
- Note: the call and put must have the same exercise price (K).

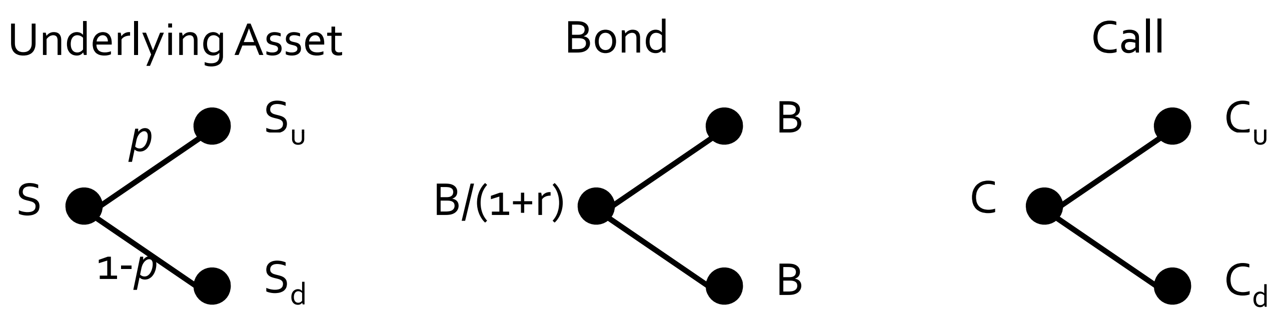
Review: value of an option



	Value of call	Value of put
Strike price (K)	Decrease	Increase
Price of underlying asset (S)	Increase	Decrease
Volatility of the underlying asset (σ)	Increase	Increase
Maturity (T)	Increase	Increase
Interest rate (r)	Increase	Decrease

Review: binomial tree

- Idea: if there are only two states of the world next period, we can price options given the underlying asset and a risk-free asset ("bond") by replication:



Review: binomial tree

- Replication:

	CF at t = 0	CF at t=1 ("up" state)	CF at t=1 ("down" state)
A shares of underlying asset	$-A \times S$	$A \times S_u$	$A \times S_d$
Bond (FV=B)	$-B/(1+r)$	B	B
Total	$-A \times S - B/(1+r)$	$A \times S_u + B$	$A \times S_d + B$
Replication	$= -C$	$= C_u$	$= C_d$

- $A = (C_u - C_d) / (S_u - S_d)$
- $B = C_u - A \times S_u$
- $C = A \times S + B/(1+r)$

Review: binomial tree

- Equivalently, we can solve for the risk-neutral probability, *q*:

$$S = \frac{qS_u + (1-q)S_d}{1+r}$$

- Then,

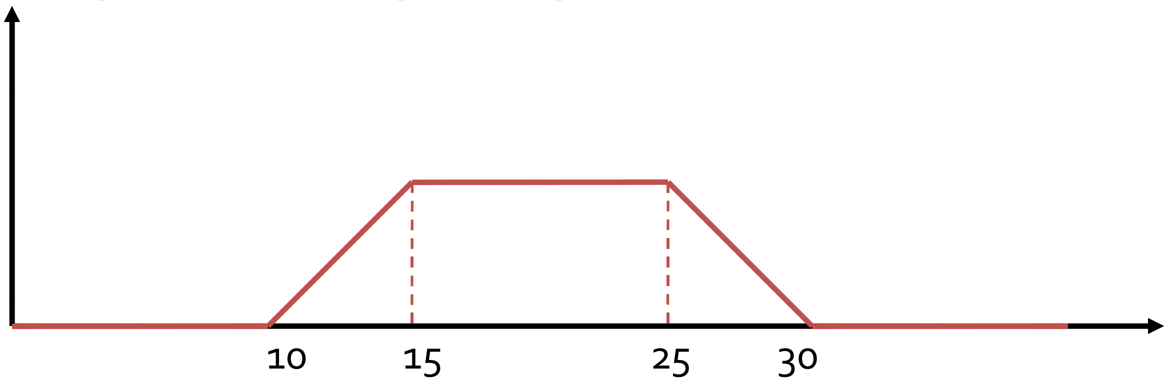
$$C = \frac{qC_u + (1-q)C_d}{1+r}$$

- Note: *q* is not related to the state probability *p*. In fact, *p* is not used in the pricing of *C*.

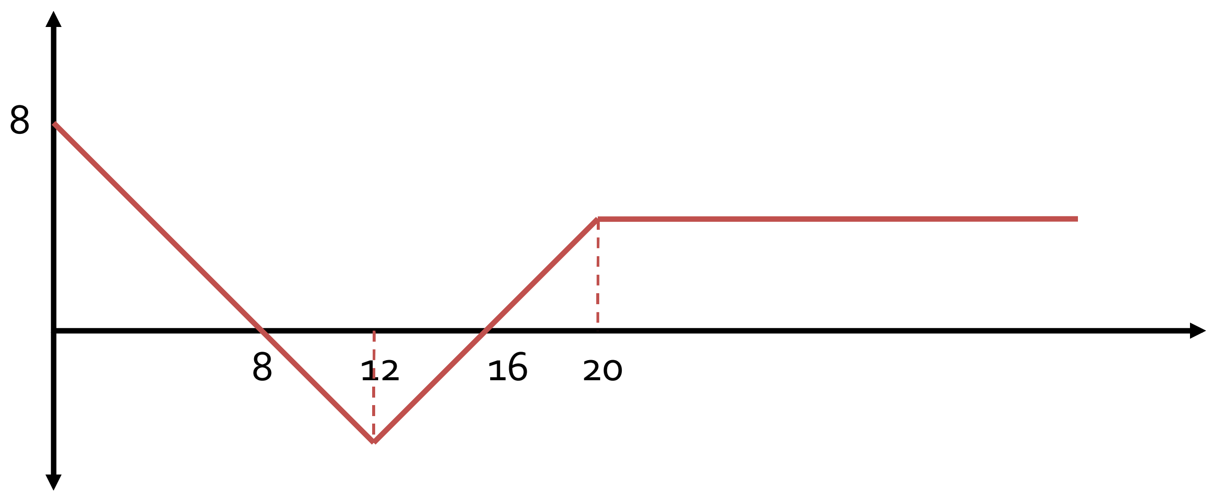
Example 1: payoff replication

- How would you replicate the following payoff profile using only call and put options?

a)



b)

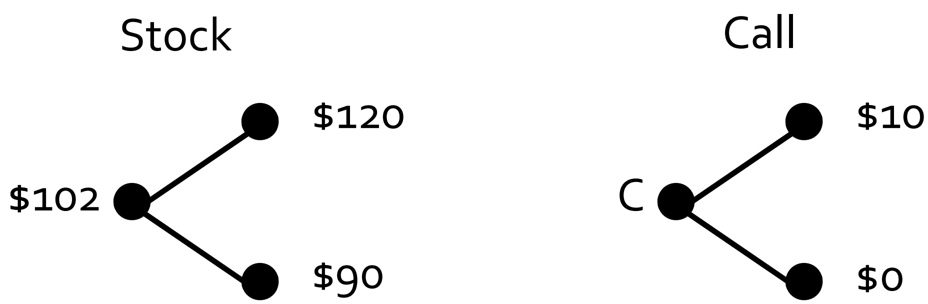


Example 1: payoff replication

- Answer:
  a) Long 1 call (K=10)
     Short 1 call (K=15)
     Short 1 call (K=25)
     Long 1 call (K=30)
  b) Long 1 put (K=8)
     Short 1 call (K=8)
     Long 2 calls (K=12)
     Short 1 call (K=20)

Example 2: Arboreal Corporation

- Arboreal Corporations stock price is currently \$102. At the end of 3 months it will be either \$120 or \$90. The 3-month spot rate is 2%. What is the value of a 3-month European call option with a strike price of \$110?



Example 2: Arboreal Corporation

- The call can be replicated with:
  - Long 1/3 stock: costs $34
  - Short bond with FV=30: costs -$30/(1+2%) = -$29.41
- The price of the call must be

$$C = 34 - 29.41 = \$4.59$$

- Alternatively, we can solve for the risk-neutral probability: $\frac{120q + 90(1-q)}{1+2\%} = 102 \Rightarrow q = 0.468$
- The price of the call is then

$$C = \frac{10(0.468) + 0(1-0.468)}{1+2\%} = \$4.59$$

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