

15.401 Recitation

7: CAPM

Learning Objectives

- Review of Concepts
 - CAPM
 - Beta and SML
 - Alpha
- Examples
 - The frontier
 - CML and SML

Review: efficient frontier

From Portfolio Choice...

- ❑ The CML is tangent to the efficient frontier at the **tangency portfolio**.
- ❑ The tangency portfolio is the portfolio of risky assets that **maximizes the Sharpe ratio**.
- ❑ The slope of the CML is the maximum Sharpe ratio.
- ❑ Rational investors always hold **a combination of the tangency portfolio and the risk-free asset**. The proportion depends on investors' risk preferences.

Review: CAPM

- ❑ Since each investor holds the **tangency portfolio** as part of his/her overall portfolio, the **market portfolio** must coincide with the tangency portfolio.
- ❑ Idea of CAPM: the contribution of a single risky asset to the risk of the market portfolio must be proportional to its risk premium.
- ❑ In other words, investors are compensated for exposure to **systematic risk**.
- ❑ **Idiosyncratic risk** is not compensated because they can be diversified away.

Review: CAPM

- A measure of an asset's systematic risk is its beta:

$$\beta_i \equiv \frac{\text{cov}(\tilde{r}_i, \tilde{r}_m)}{\text{var}(\tilde{r}_m)} = \frac{\rho_{im} \sigma_i \sigma_m}{\sigma_m^2} = \rho_{im} \frac{\sigma_i}{\sigma_m}$$

- Core result of CAPM:

$$\bar{r}_i = r_f + \beta_i (\bar{r}_m - r_f)$$

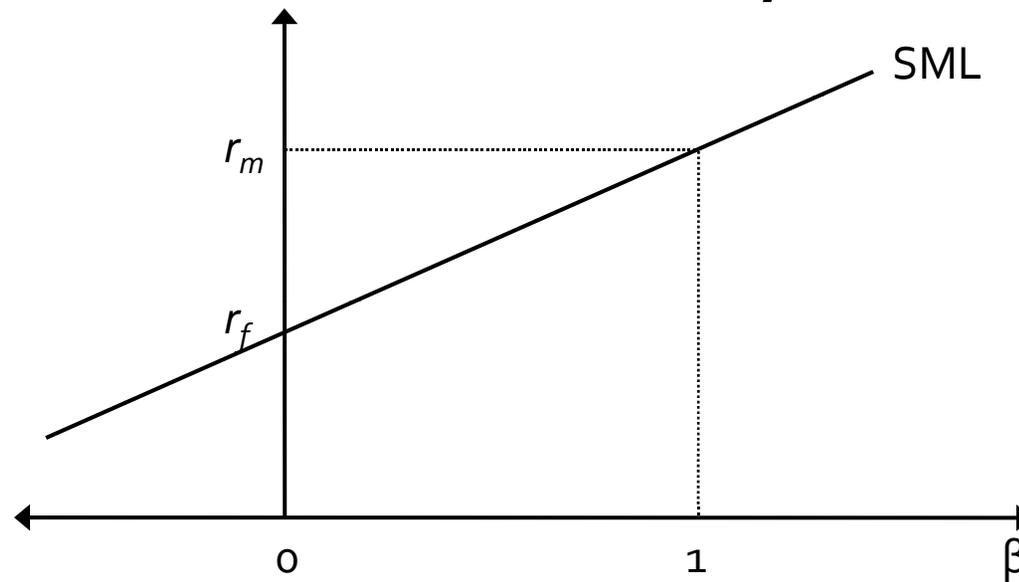
- Note:

Market portfolio: $\beta_m = 1 \Rightarrow \bar{r}_m = r_f + (\bar{r}_m - r_f)$

Risk-free portfolio: $\beta_f = 0 \Rightarrow \bar{r}_f = r_f + 0 \cdot (\bar{r}_m - r_f)$

Review: SML

- Graph of $\bar{r}_i = r_f + \beta_i(\bar{r}_m - r_f)$ in (beta, return) space is a straight line called the **Security Market Line**:



- If CAPM holds, every asset must be on the SML.

Review: testing CAPM

- ❑ CAPM does not hold exactly
- ❑ The regression

$$r_i = \alpha_i + \beta_i(r_m - r_f) + \varepsilon_i$$

often give nonzero **alpha**.

- ❑ CAPM requires alpha to be zero for all assets
- ❑ CAPM may fail if factors other than beta affect asset returns, such as
 - Fama-French factors: market (beta), size, and book-to-market

Review: portfolio beta and alpha

□ The beta of a portfolio is

$$\beta_p = \sum_{i=1}^N w_i \beta_i$$

□ The alpha of a portfolio is

$$\alpha_p = \sum_{i=1}^N w_i \alpha_i$$

Example 1: the frontier

- The risk-free rate is 6%, the expected return on the market portfolio is 14%, and the standard deviation of the return on the market portfolio is 25%. Consider a portfolio with expected return of 16% and assume that it is on the efficient frontier.
 - a. What is the beta of this portfolio?
 - b. What is the standard deviation of its return?
 - c. What is its correlation with the market return?

Example 1: the frontier

□ Answer:

a.
$$\bar{r}_p = r_f + \beta_p (\bar{r}_m - r_f)$$

$$0.16 = 0.06 + \beta_p (0.14 - 0.06)$$

$$\beta_p = 1.25$$

b. Since the portfolio is on the efficient frontier, it is a combination of the risk-free asset (w) and the market portfolio ($1-w$):

$$0.16 = 0.06w + 0.14(1 - w)$$

$$w = -0.25$$

Example 1: the frontier

□ Answer:

b. The standard deviation of the portfolio is

$$\sigma_p = 1.25 \cdot \sigma_m = 1.25 \cdot 25\% = 31.25\%$$

c. The beta of the portfolio is

$$\beta_i = \rho_{im} \frac{\sigma_i}{\sigma_m}$$

$$1.25 = \rho_{im} \frac{0.3125}{0.25}$$

$$\rho_{im} = 1$$

Example 2: CML and SML

- Using the properties of the capital market line (CML) and the security market line (SML), determine which of the following scenarios are consistent or inconsistent with the CAPM. Explain your answers.
- Let A and B denote arbitrary securities while F and M represent the riskless asset and the market portfolio respectively.

Example 2: CML and SML

□ Scenario I:

Security	E[R]	β
A	25%	0.8
B	15%	1.2

□ Answer: **inconsistent**

Higher beta requires higher expected return

Example 2: CML and SML

□ Scenario II:

Security	$E[R]$	$\sigma[R]$
A	25%	30%
M	15%	30%

□ Answer: **inconsistent**

A lies above the CML, which means that the market portfolio is inefficient.

Example 2: CML and SML

□ Scenario III:

Security	$E[R]$	$\sigma[R]$
A	25%	55%
F	5%	0%
M	15%	30%

□ Answer: **inconsistent**

A lies above the CML, which means that the market portfolio is inefficient.

Example 2: CML and SML

□ Scenario IV:

Security	E[R]	β
A	20%	1.5
F	5%	0
M	15%	1.0

□ Answer: **consistent**

Portfolio A lies on the SML

Example 2: CML and SML

□ Scenario V:

Security	E[R]	β
A	35%	2.0
M	15%	1.0

□ Answer: **inconsistent**

The implied risk-free rate would be negative if A lies on the SML.

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