18.01 Exam 3

Problem 1. (20 pts) Evaluate the following integrals

$a) \int_0^2 \frac{xdx}{(1+x^2)^2}$

$b) \int_{-\pi/2}^{\pi/2} \sin^6 x \cos x dx$

Problem 2. (20 pts.) Find the following approximations to

$$\int_0^{\pi/2} \cos x dx$$

(Do not give a numerical approximation to square roots; leave them alone.)

a) Using the upper Riemann sum with two intervals

b) Using the trapezoidal rule with two intervals

c) Using Simpson's rule with two intervals

Problem 3. (20 points) Find the volume of the solid of revolution formed by revolving the *y*-axis the region enclosed by

$$y=\cos(x^2)$$

and the *x*-axis (central hump, only).

Problem 4. (20 points) Students studying for an exam get *x* hours of sleep in the two days leading up to the exam, where *x* is the range $0 \le x \le a$. The numbers of students who got between x_1 and x_2 hours of sleep in given by

$$\int_{x_1}^{x_2} cxdx; 0 \le x_1 \le x_2 \le a$$

a) What fraction o the student got less than a/2 hours of sleep?

b) Their scores are proportional to the amount of sleep they got: $S(x) = 100\ (x/a)$. Find the (correctly weighted) average score in the class.

Problem 5. (20 points) Let

$$F(x)=\int_0^x \sqrt{t}\sin t dt$$

a) Find *F′(x)* for *x > 0* identify the points *a > 0 F′(a) = 0*

b) Decide whether F has a local maximum at the smallest critical point a > 0 that you found in part (a) by evaluating *F″.*

c) Say whether *F(x)* is positive, negative or zero at each of the following points, and give a reason in each case.

$i)\, x=0$

$ii)\, x=\pi$

$iii)\, x=2\pi$

d) Use a change of the variable to express $G(x)=\int_0^x u^2\sin(u^2)du$ in terms of *F.*