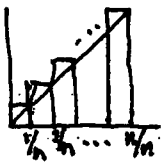


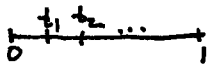
EXAM 3 PRACTICE: SOLUTIONS: FALL 2006

1) a) $\int_0^1 \frac{x dx}{\sqrt{1+3x^2}} = \frac{1}{3} (1+3x^2)^{1/2} \Big|_0^1 = \frac{1}{3} (2-1) = \frac{1}{3}$

b) $\int_{\pi/3}^{\pi/2} \cos^3 x \sin 2x dx = \int_{\pi/3}^{\pi/2} 2 \cos^4 x \sin x dx$
 $= -\frac{2}{5} \cos^5 x \Big|_{\pi/3}^{\pi/2} = -\frac{2}{5} \left[0 - \left(\frac{1}{2}\right)^5 \right]$
 $= \frac{1}{5 \cdot 16}$

2)  Upper sum is sum of circums. rectangles
 $= \sum_{i=1}^n \left(\frac{1}{n}\right) \frac{1}{n}$
 $= \frac{1}{n^2} \sum_{i=1}^n i = \frac{n^2+n}{2n^2} = \frac{1}{2} + \frac{1}{2n}$

$\lim_{n \rightarrow \infty} \int_0^1 x dx = \frac{1}{2}$

3)  Divide interval into n equal subintervals, length Δt.

* first deposited in the time interval grows by year-end to $k \Delta t e^{r(1-t_i)}$

Total at year-end $\approx \sum_{i=1}^n k e^{r(1-t_i)} \Delta t$

As $n \rightarrow \infty$, this $\rightarrow \int_0^1 k e^{r(1-t)} dt$

[or can make argument using dt instead of Δt: replace Δt by dt in (*), then pass directly to the last line]

4) a) On $[0, 1]$, $\sin t < 1$

So $\int_0^1 \sqrt{3+\sin t} dt < \int_0^1 \sqrt{4} dt = 2$.

b) $F'(x) = \sqrt{3+\sin x}$ (2nd F.T.)

$F''(x) = \frac{\cos x}{2\sqrt{3+\sin x}} > 0$ on $[0, 1]$

$\therefore$ convex (concave up).

c) Set $u = 2t$, so $du = 2 dt$

$\int_1^2 \sqrt{3+\sin 2t} dt = \int_2^4 \sqrt{3+\sin u} \cdot \frac{1}{2} du$

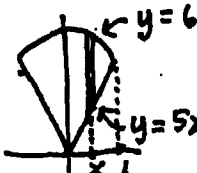
$= \frac{1}{2} F(u) \Big|_2^4 = \frac{1}{2} [F(4) - F(2)]$

5) $\int_0^0 f(t) dt = 0 = e^{2 \cdot 0} \cos 0 + C$
 $\therefore C = -1$

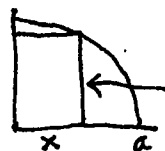
By 2nd F.T.:

$f(x) = D(e^{2x} \cos x - 1)$
 $= 2e^{2x} \cos x - e^{2x} \sin x$

6)  horizontal slice $\rightarrow$
 $\int_0^{\sqrt{a}} (\pi a^2 - \pi x^2) dy = \pi \int_0^{\sqrt{a}} (a^2 - y^4) dy$
 $= \pi \left(a^2 y - \frac{y^5}{5} \right) \Big|_0^{\sqrt{a}} = \pi \left[a^{5/2} - \frac{1}{5} a^{5/2} \right]$
 $= \pi \cdot \frac{4}{5} a^{5/2}$

7)  The strip rotated has volume $2\pi x(6-x^2-5x) dx$

Volume $= 2\pi \int_0^1 x(6-x^2-5x) dx$
 $= 2\pi \left[3x^2 - \frac{1}{4}x^4 - \frac{5}{3}x^3 \right]_0^1$
 $= 2\pi \left[3 - \frac{1}{4} - \frac{5}{3} \right] = 2\pi \cdot \frac{17}{12}$

8)  curve: $y = \sqrt{a^2-x^2}$
 $\sqrt{a^2-x^2}$
 $\text{Area} = \int_0^a \sqrt{a^2-x^2} dx$

Average area $= \frac{1}{a} \int_0^a x \sqrt{a^2-x^2} dx$
 $= \frac{1}{a} (a^2-x^2)^{3/2} \Big|_0^a = \frac{1}{3} (a^2)^{3/2} = \frac{a^2}{3}$

9)

x	0	$\pi/4$	$\pi/2$
$\sin x$	0	$1/\sqrt{2}$	1
$\sin^6 x$	0	$1/8$	1

 $\Delta x = \pi/4$
 SIMPSON
 $\int_0^{\pi/2} \sin^6 x dx \approx \frac{\pi/4}{3} \left(0 + 4 \cdot \frac{1}{8} + 1 \right)$
 TRAPEZOIDAL
 $\approx \frac{\pi}{4} \left(\frac{1}{2} \cdot 0 + \frac{1}{8} + \frac{1}{2} \right)$
 $= \frac{\pi}{4} \cdot \frac{5}{8} = \frac{5\pi}{32}$
 $= \frac{\pi}{12} \left(\frac{3}{2} \right) = \frac{\pi}{8}$